

GOVERNMENT BUDGET AND FISCAL POLICY

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Outline

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- 6) Debt Accumulation
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Question 1: Is the U.S. in Trouble in Terms of Debt?

How can we determine this for any other country?



Source: <https://www.doge.gov/savings>

- In 2023, U.S. debt-service payments reached a historical high.
- In fiscal year 2024, federal spending exceeded revenue by a large margin (deficit around \$1.8T).
- As of early 2025, U.S. public debt exceeded \$36T and was above 120% of GDP in gross terms.
- Policy question: under realistic $r - g$ and primary-balance paths, are current debt dynamics sustainable?

Question 2: How Much Debt Is Too Much?



Sources: EIU; Foreign Policy

- There is no single debt threshold: risk depends on $r - g$, maturity, credibility, and market beliefs.

Question 3: Why Is Sovereign Debt Important?

- Sovereign debt has existed for centuries and remains central to macroeconomics and finance.
- It is one of the largest global financial asset classes.

Sovereign Debt as a Share of All Financial Assets

	Listed in London				London and Some Foreign			World			
	1853 ^a	1873 ^a	1893 ^a	1913 ^a	1933 ^a	1950 ^a	1950 ^b	1950 ^c	1978 ^c	1990 ^d	2010 ^d
Total Assets	£1.2b	£2.3b	£4.9b	£11.2b	£18.5b	£15.2b	£14.3b	--	--	\$54t	\$212t
Public Debt	76%	59%	39%	35%	59%	78%	60%	22%	11%	17%	19%
Of which: UK	70%	38%	18%	14%	38%	66%	57%	--	--	--	--
Foreign & Colonial	6%	21%	21%	21%	21%	11%	3%	--	--	--	--

^aData from *Stock Exchange Official Intelligence* as reported in Michie (2001) Tables 3.2 and 5.1. All securities at face value. Data refer to securities listed on the London Stock Exchange, except for 1933 and 1950 which includes foreign and colonial public sector securities listed abroad.

^bData from *Stock Exchange Official Intelligence* as reported in Michie (2001) Table 8.1. All securities at market value. Data refer to securities listed on the London Stock Exchange, plus foreign and colonial public sector securities listed abroad.

^cData from Goldsmith (1985) Table 1 on share of government debt in financial assets, all measured at market values.

^dData from Roxburgh, Lund and Piotrowski (2011) Exhibit E1 on public debt securities at face value, relative to other debt at face value and equities at market value.

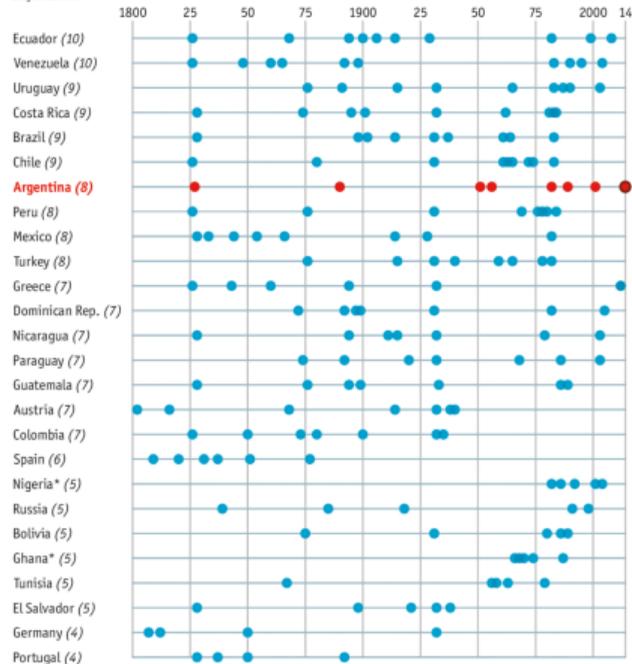
Source: Tomz and Wright (2013), as used in 'fiscal2026.tex'.

Question 4: Are Sovereign Debt Crises Rare?

External sovereign defaults since 1800

Selected countries (number of defaults)

July 31st 2014



Source: Carmen Reinhart and Kenneth Rogoff

*Data from 1960

Economist.com/graphicdetail

- Between 1820 and 2012, 251 defaults by 107 entities.
- Most frequent defaulters: Ecuador, Mexico, Uruguay, Venezuela (+ 8 episodes each); serial defaulters (Reinhart and Rogoff, 2004).
- Largest defaults: Greece 2012 - \$ 261bn, Argentina 2002 - USD132bn.
- Unconditional probability of default is 1.8% per year:
 - Extremely sensitive to definition of default, particularly how different episodes are aggregated.
 - Arteta and Hale (2008): probability of default of 13 % per year.

Government Debt Dynamics and Ricardian Equivalence

Introduction to Fiscal Policy

- **Fiscal policy** involves government decisions regarding levels and composition of **taxation** and **public spending**.
- **Direct effect:** In contrast to monetary policy (interest rates, money supply), fiscal policy directly impacts aggregate demand through spending (G) and taxes (T).
- **Political process:** Fiscal decisions are influenced by political processes, making rapid policy adjustments challenging compared to monetary policy.
- **Consensus:** Stabilization policies are better managed by monetary authorities.
 - Quick implementation
 - Less political interference
- **Importance of Fiscal Policy:** Even if monetary policy is optimal for short-term stabilization, fiscal policy profoundly influences long-run economic growth, income distribution, and efficiency. For instance, distortionary taxes may discourage work or investment, while well-structured government spending (e.g., infrastructure, education) can boost economic growth.

Government Debt Dynamics

Two periods

- **Period 1:** The government finances spending through either taxes or debt

$$G_1 = T_1 + B$$

- **Period 2:** The government finances spending through taxes

$$G_2 + (1 + r)B = T_2$$

Multiple Periods (N)

- Generalize this into a multi-period discrete model for $t = 1, 2, \dots, N$:

$$G_t + (1 + r)B_{t-1} = T_t + B_t$$

Continuous time

$$G_t + (r)B_t = T_t + \dot{B}_t \quad \longrightarrow \quad \dot{B}_t = G_t + rB_t - T_t \quad (1)$$

- \dot{B}_t is the change in debt at time t .
- Debt accumulates whenever government spending (+ interest payments) exceeds taxes.

Government Debt Dynamics (+)

- Note that $B(t)$ denotes the **level (or stock)** of government debt outstanding at time t .
- $B(t)$ represents the **stock of accumulated debt** that the government owes at precisely moment t .
- $\dot{B}(t)$, which is the derivative of $B(t)$ with respect to time, represents the **change (or flow)** of debt at time t .
 - If $\dot{B}(t) > 0$, debt increases (the government is borrowing more).
 - If $\dot{B}(t) < 0$, debt decreases (the government is repaying debt).
- Thus, the term $rB(t)$ specifically means the **interest payments** on the existing level (stock) of debt at time t .

Government Budget Constraint

From $\dot{B}(t) = G(t) - T(t) + r(t)B(t)$, define cumulative discounting as

$$R(t) \equiv \int_0^t r(\tau) d\tau.$$

Then the intertemporal government budget constraint is

$$\int_0^\infty e^{-R(t)} G(t) dt \leq -B(0) + \int_0^\infty e^{-R(t)} T(t) dt \quad (2)$$

- Intuition: discounted spending cannot exceed discounted taxes net of initial debt.
- Equivalent primary-surplus form:

$$\int_0^\infty e^{-R(t)} [T(t) - G(t)] dt \geq B(0).$$

- Borrowing can shift taxes over time, but cannot eliminate the need to finance spending.

No-Ponzi Condition

The intertemporal budget restriction is equivalent to

$$\lim_{s \rightarrow \infty} e^{-R(s)} B(s) \leq 0 \quad (3)$$

- Intuition: the present value of outstanding debt cannot stay strictly positive forever.
- Debt may rise over time, but not faster than the economy can service in present-value terms.
- This is the key solvency condition behind fiscal sustainability analysis.

Transversality Logic

From $\dot{B}(t) = G(t) - T(t) + r(t)B(t)$, for any horizon S :

$$B(0) = \int_0^S e^{-R(t)} [T(t) - G(t)] dt + e^{-R(S)} B(S)$$

- The last term is **discounted terminal debt**.
- As $S \rightarrow \infty$, solvency requires this terminal term not remain strictly positive.
- That gives the transversality/no-Ponzi restriction:

$$\lim_{S \rightarrow \infty} e^{-R(S)} B(S) \leq 0.$$

Economic Intuition: No-Ponzi vs Transversality

- **No-Ponzi interpretation:** the government cannot fund spending forever by issuing debt that is never repaid in present-value terms.
- **Transversality interpretation:** terminal discounted debt cannot be a positive "free lunch" in equilibrium.
- If this failed, current generations could consume more while leaving an explosive claim to the future.
- In realistic economies, lenders and market pricing discipline this behavior, so fiscal plans must eventually be backed by primary surpluses.

Ponzi Games and Dynamic Inefficiency

- In standard environments, governments must satisfy (2) and (3).
- A theoretical exception is a dynamically inefficient economy (roughly, $r < g$), where perpetual rollover may be feasible.
- In practical policy analysis for advanced economies, this is not a reliable strategy for long-run fiscal planning.
- Therefore the conservative benchmark is: eventually, primary surpluses must support debt sustainability.

Government: It is all about tax

- **How does the government finance itself?**
 - Current tax income.
 - Borrowing against future tax income.
 - Borrowing today implies higher taxes later for repayment.
 - Taxing more today reduces private resources now.
- **How does this choice affect household consumption?**
- **Ricardian Equivalence:** It does not matter (Barro, 1974).
- Rational, forward-looking households save more to offset expected future taxes if the government borrows.
- **How?**

Ricardian Equivalence: Notation (Continuous Time)

- $C(t)$: private consumption at time t .
- $Y(t)$: household income (output/endowment) at time t .
- $T(t)$: lump-sum taxes paid by households at time t .
- $G(t)$: government purchases at time t .
- $K(0)$: initial private assets; $B(0)$: initial government debt held by households.
- $e^{-R(t)}$: discount factor, where $R(t) = \int_0^t r(s) ds$.
- $r(s)$: real interest rate at date s .

Ricardian Equivalence: Formal Derivation

Household intertemporal budget constraint:

$$\int_0^{\infty} e^{-R(t)} C(t) dt \leq K(0) + B(0) + \int_0^{\infty} e^{-R(t)} [Y(t) - T(t)] dt$$

Assume the government satisfies its intertemporal budget constraint with equality (under a no-Ponzi condition):

$$\int_0^{\infty} e^{-R(t)} T(t) dt = B(0) + \int_0^{\infty} e^{-R(t)} G(t) dt$$

Substitute into the household constraint:

$$\int_0^{\infty} e^{-R(t)} C(t) dt \leq K(0) + \int_0^{\infty} e^{-R(t)} Y(t) dt - \int_0^{\infty} e^{-R(t)} G(t) dt$$

Tax timing disappears; only the path of government purchases matters for real allocations.

Ricardian Equivalence: 2 Periods

Household Budget Constraint:

Period 1: Household income is Y_1 , consumption is C_1 , taxes are T_1 , and saving is S :

$$Y_1 - T_1 = C_1 + S$$

Period 2: Household receives income Y_2 , pays taxes T_2 , consumes C_2 , and earns interest on savings:

$$Y_2 - T_2 + (1 + r)S = C_2$$

Combining both periods we can get:

$$C_1 + \frac{C_2}{1 + r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1 + r} \quad (4)$$

Government Budget Constraint:

Period 1: Government finances spending G_1 via taxes T_1 or debt issuance B :

$$G_1 = T_1 + B$$

Period 2: Debt and interest must be paid from taxes collected:

$$G_2 + (1 + r)B = T_2$$

Ricardian Equivalence: 2 Periods: Implications

Ricardian Equivalence in Household Decision-Making:

- Using the government constraints, households see clearly that a decrease in taxes today ($T_1 \downarrow$) implies an increase in taxes tomorrow ($T_2 \uparrow$).
- Thus, household lifetime resources remain unchanged:

$$\begin{aligned}(Y_1 - T_1) + \frac{Y_2 - T_2}{1 + r} &= (Y_1 - T_1) + \frac{Y_2 - [G_2 + (1 + r)(G_1 - T_1)]}{1 + r} \\ &= Y_1 + \frac{Y_2 - G_2 - (1 + r)G_1}{1 + r}\end{aligned}$$

Conditions for Ricardian Equivalence:

- Lump-sum, non-distortionary taxes
- Perfect capital markets (households borrow/save at government interest rates)
- Rational, forward-looking households

Ricardian Equivalence: Infinite-Horizon Intuition

Using (16), the household's lifetime resource constraint becomes

$$\int_0^{\infty} e^{-R(t)} C(t) dt \leq K(0) + \int_0^{\infty} e^{-R(t)} [Y(t) - G(t)] dt \quad (5)$$

- Debt $B(0)$ and taxes $T(t)$ no longer appear separately after substitution.
- A bond-financed tax cut today is offset by future taxes in present-value terms.
- Therefore, under the Ricardian assumptions, financing mix (tax now vs debt now) does not change aggregate consumption paths.

Ricardian Equivalence: Why It Fails in Practice

- **Finite lifetimes and population turnover:** part of future tax burdens can fall on future cohorts, so current bond holders may treat debt as net wealth.
- **Liquidity constraints:** households that cannot borrow cheaply consume more when current taxes are cut.
- **Precautionary saving with uncertainty:** future taxes are often state-contingent, so current tax cuts can alter risk exposure, not just timing.
- **Distortionary taxation:** when taxes affect labor supply, saving, and investment, tax timing changes real behavior.
- **Empirical benchmark:** the 2008-2009 U.S. tax cuts increased consumption for many households, inconsistent with strict Ricardian neutrality.

Tax Smoothing: Optimal Taxation

Cost of Distortionary Taxes: Reasons

Most taxes are **distortionary**—they alter people's behavior and economic decisions. Specifically:

- **Labour Income Taxes:**

- Discourage work effort or reduce the incentive to supply labor (workers may choose leisure over additional taxed income).

- **Capital Income Taxes:**

- Discourage saving and investment by reducing the after-tax returns on capital.

- **Value-Added Taxes (VAT) and Consumption Taxes:**

- Distort consumption decisions by changing relative prices of goods and services, affecting consumer choices.

- These distortions introduce economic inefficiencies, reducing overall welfare and potentially hindering economic growth.

Cost of Distortionary Taxes: Optimal Taxation

- Government finances predetermined spending G_1, G_2 .
- Finances via taxes T_t or borrowing at interest rate r .
- Government budget constraint:

$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

- The optimal tax rate satisfies:

Marginal social cost of taxation = Marginal social benefit of taxation

- **Resource Constraint** (Economy-wide): (zero net export and no investment)

$$Y_t = C_t^{private} + G_t + \zeta_t$$

- Where ζ_t is the distortionary tax cost:

$$\zeta_t = Y_t f\left(\frac{T_t}{Y_t}\right)$$

Cost of Distortionary Taxes: Optimal Taxation

$$\zeta_t = Y_t f\left(\frac{T_t}{Y_t}\right), \quad f(0) = 0, \quad f'(\cdot) > 0, \quad f''(\cdot) < 0$$

- ζ_t represents the **total cost** (in terms of lost output or welfare) due to **distortionary taxes** at time t .
- The ratio $\frac{T_t}{Y_t}$ is the **tax rate** expressed as a proportion of the economy's total output or income. This normalized tax rate is critical because distortionary costs typically depend on **how high** taxes are relative to the economy's overall size.
- $f\left(\frac{T_t}{Y_t}\right)$ represents the distortionary cost **per unit of output** as a function of the tax rate.
- Thus, the distortionary cost per unit of output is scaled by total output Y_t to get **total distortionary costs** in absolute terms:

Distortionary cost per unit output \times Total output = Total distortionary cost

- Higher $\frac{T_t}{Y_t}$ (taxes represent a larger share of output) \rightarrow stronger economic distortions.
- Lower $\frac{T_t}{Y_t}$ \rightarrow milder distortions.

Cost of Distortionary Taxes: Optimal Taxation

- The government's problem can be expressed as:

$$\min_{T_1, T_2} \left[Y_1 f \left(\frac{T_1}{Y_1} \right) + \frac{Y_2}{1+r} f \left(\frac{T_2}{Y_2} \right) \right]$$

- subject to its budget constraint:

$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

- The government seeks to minimize the total discounted distortionary cost across both periods subject to the budget constraint above.

Cost of Distortionary Taxes: Optimal Taxation

- Define the Lagrangian and take the FOC with respect to T_1 :

$$\frac{\partial \mathcal{L}}{\partial T_1} = Y_1 f' \left(\frac{T_1}{Y_1} \right) \frac{1}{Y_1} - \lambda = 0$$

- Simplifies explicitly to:

$$f' \left(\frac{T_1}{Y_1} \right) = \lambda \tag{6}$$

- With respect to T_2 :

$$\frac{\partial \mathcal{L}}{\partial T_2} = \frac{Y_2}{1+r} f' \left(\frac{T_2}{Y_2} \right) \frac{1}{Y_2} - \frac{\lambda}{1+r} = 0$$

- Again simplifying explicitly:

$$f' \left(\frac{T_2}{Y_2} \right) = \lambda \tag{7}$$

Cost of Distortionary Taxes: Optimal Taxation

- From the FOCs (equations 6 and 7), we directly see that:

$$f' \left(\frac{T_1}{Y_1} \right) = f' \left(\frac{T_2}{Y_2} \right) = \lambda \quad (8)$$

- **Marginal distortionary cost** (deadweight loss per additional unit of revenue raised) must be equalised across both periods.
- At optimum, no further gains in efficiency are possible through intertemporal reallocation.
- This explicitly ensures the **efficiency** of intertemporal taxation.
- Since f is strictly convex, combining (6) and (7) implies a constant **tax rate** (tax as a share of output) across periods:

$$\boxed{\frac{T_1}{Y_1} = \frac{T_2}{Y_2}} \quad (9)$$

Optimal Taxation: Implications

$$\frac{T_1}{Y_1} = \frac{T_2}{Y_2}$$

- The core result of tax smoothing theory highlights that a rational, welfare-maximising government aims to keep tax rates stable across periods.
- Frequent and significant variations in taxes are economically costly and inefficient.
- Optimal fiscal policy usually involves stable, predictable taxation rates rather than volatile short-term adjustments.
- Policymakers thus strive for consistent, stable taxation frameworks, which help promote economic efficiency, long-term growth, and fiscal sustainability.

**But how does the government manage temporary and unexpected expenditures?
Like wars and recessions.**

Optimal Taxation: Implications

- If governments faced temporary expenditure shocks and tried to finance these entirely through current taxes, this would cause substantial tax increases, sharply raising marginal distortion costs.
- To avoid such costly disruptions, the optimal strategy for governments is borrowing.

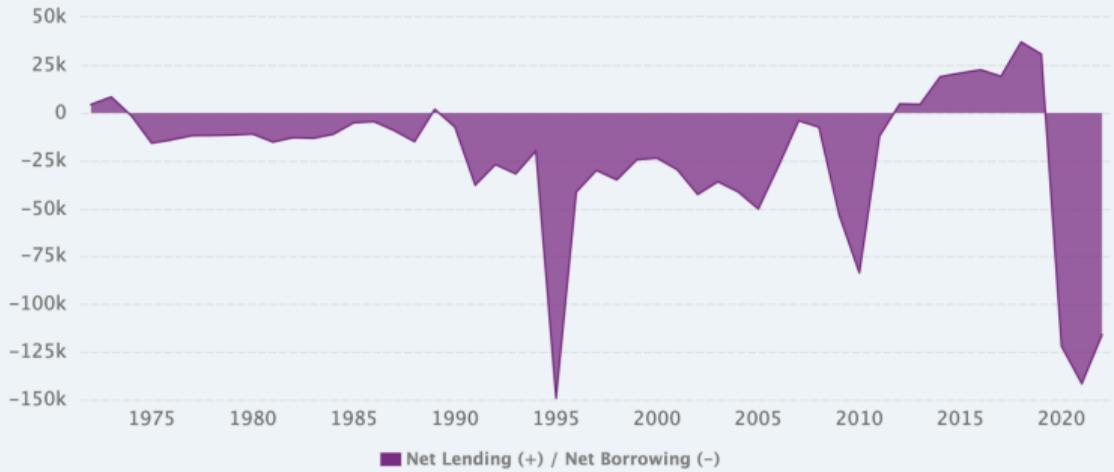
Why Borrowing?

- Borrowing allows governments to smooth out temporary spikes in expenditures over several periods, keeping tax rates stable and avoiding dramatic short-run tax increases.
- By spreading out repayment over the future (when expenditure returns to normal), borrowing significantly reduces total economic inefficiencies.
- **How does this work in practice?**

Debt and Deficits

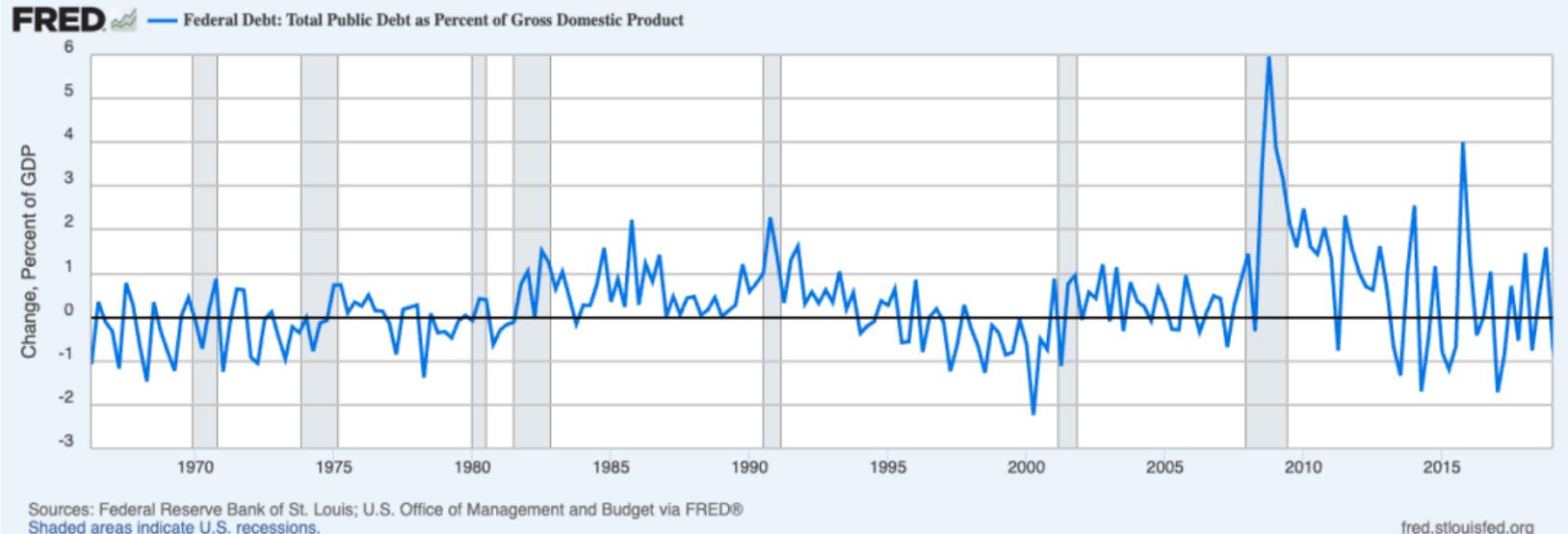
View Germany's Germany DE: Net Lending (+) / Net Borrowing (-) from 1972 to 2022 in the chart:

max 1y 5y 10y area December 1, 2011 December 1, 2022 Apply Get this data

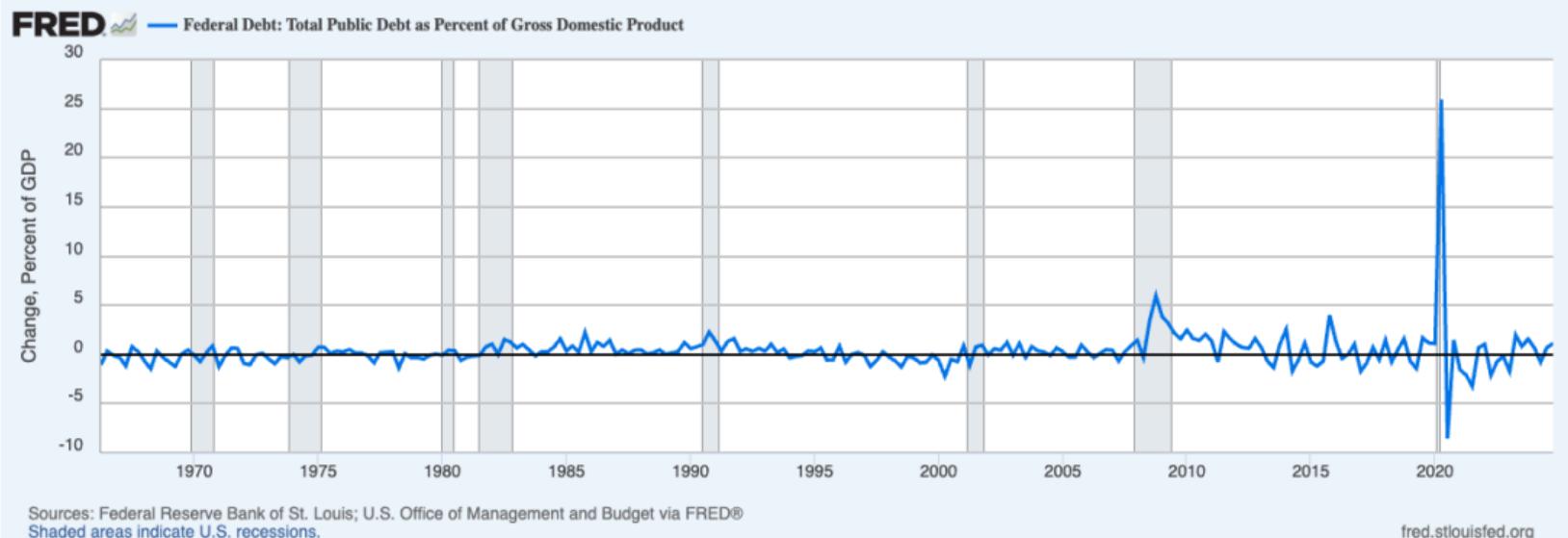


SOURCE: WWW.CEICDATA.COM | World Bank

Debt and Deficits



Debt and Deficits



Tax Smoothing Under Uncertainty

- With uncertainty, optimal policy smooths expected marginal distortion over time:

$$f' \left(\frac{T_t}{Y_t} \right) = E_t \left[f' \left(\frac{T_{t+1}}{Y_{t+1}} \right) \right]$$

- If distortion costs are quadratic, this simplifies to:

$$\frac{T_t}{Y_t} = E_t \left[\frac{T_{t+1}}{Y_{t+1}} \right]$$

- Intuition: tax rates move with fiscal news; temporary spending shocks are mostly absorbed by deficits and surpluses.

Jensen Inequality Intuition (Key Pedagogy)

Let $\tau_t \equiv T_t/Y_t$ and distortion cost per unit output be $f(\tau_t)$ with $f'' > 0$ (convex).

$$E[f(\tau)] \geq f(E[\tau]) \quad (\text{Jensen inequality})$$

- If tax rates are volatile, expected distortion cost is higher.
- Holding average tax rate fixed, smoother taxes lower expected welfare loss.
- This is the deep intuition behind tax smoothing under uncertainty.

Jensen Example: Same Mean, Higher Expected Cost

Take $f(\tau) = \tau^2$, where $\tau = T/Y$ is the tax rate.

- **Smooth policy:** $\tau = 0.20$ in every state

$$E[f(\tau)] = 0.20^2 = 0.04$$

- **Volatile policy:** $\tau = 0.10$ or 0.30 with equal probability

$$E[f(\tau)] = \frac{1}{2}(0.10^2 + 0.30^2) = 0.05$$

Same average tax rate (0.20), but higher expected distortion under volatility.

Debt Arithmetic and Sustainability

Not temporary but persistent

- While tax smoothing can justify short-run deficits (e.g., wars, recessions), it does not naturally explain **why some countries continually run deficits even in good economic times.**
- Persistent deficits imply that the government is continually borrowing, adding to debt levels every year, rather than just borrowing temporarily.
- Persistent deficits can reflect structural problems or political incentives, rather than optimal fiscal strategies.
- For example, the U.S. has run large deficits consistently over several decades.
 - **Auerbach & Gale (2017, Brookings)** estimate an infinite-horizon fiscal gap of about **5.39% of GDP**; using CMS actuaries' higher health-cost assumptions raises the gap by about **2.3% of GDP**, to roughly **7.7% of GDP**.
 - If revenues are benchmarked at about **18% of GDP**, a **7.7% of GDP** gap implies an illustrative tax increase of about **43% (7.7/18)**.

Source: Auerbach & Gale (2017), Brookings:

<https://www.brookings.edu/articles/the-fiscal-outlook-at-the-beginning-of-the-trump-administration/>

From Fiscal Gap to Political Choice (Zucman View)

- Fiscal arithmetic is mechanical: with $F = 0.077Y$ and $Y \approx \$30$ trillion, adjustment is **\$2.31** trillion/year (about **\$6,800** per person).
- But incidence is political: governments choose **who pays** and **what is protected**.
- Budget identity:

$$\Delta T^L + \Delta T^K + \Delta G = F$$

where ΔG is spending restraint, ΔT^L labor/consumption-side taxation, and ΔT^K capital/wealth/profit taxation.

- Real trade-off: cutting G can mean a smaller social state (health, education, transfers); raising T^L can load adjustment onto workers and middle incomes; raising T^K with enforcement targets top wealth/profits.
- **Zucman-side approach:** protect core social spending and shift more adjustment to progressive capital/wealth/profit taxation plus anti-evasion enforcement.



Gabriel Zucman

Policy implication: not just efficiency, but distribution and state capacity.

Research link: <https://www.taxobservatory.eu>

Public Debt, Deficit, and Beyond

- All variables are expressed in real terms (goods).
- Define clearly:
 - Government spending in period t : G_t
 - Real interest rate: r_t
 - Taxes in period t : T_t
- Primary deficit in period t : $G_t - T_t$
 - If $G_t - T_t > 0$, the government runs a deficit and must borrow; denote debt by B_t .
- Law of motion of government debt:

$$B_t = (1 + r_t) B_{t-1} + G_t - T_t$$

- Then, we can write the budget deficit (\mathcal{D}) in year t as

$$Deficit(\mathcal{D}_t) = \Delta B_t = B_t - B_{t-1} = \underbrace{r_t B_{t-1}}_{\text{Interest Payments}} + \underbrace{G_t - T_t}_{\text{Primary Deficit}}$$

Public Debt, Deficit and Beyond

$$\mathcal{D}_t = \Delta B_t = B_t - B_{t-1} = r_t B_{t-1} + G_t - T_t$$

- The total deficit, which is equal to the change in government debt ΔB_t , is equal to the sum of interest payments and the primary deficit $G_t - T_t$
- B_{t-1} is government debt at the end of year $t - 1$, or, equivalently, at the beginning of year t ;
- r is the real interest rate, which we shall assume to be constant here.
- Thus, rB_{t-1} equals the real interest payments on the government debt in year t .

Do not confuse the words **deficit** and **debt**. Debt is a stock— what the government owes as a result of past deficits. The deficit is a flow—how much the government borrows during a given year.

A Fiscal Policy: Decreasing Tax in Year 1

- Initial condition (year 0): Balanced budget ($G = T$), zero debt ($B_0 = 0$).
- Policy intervention:
 - Year 1: Taxes reduced temporarily by 1 unit; spending remains constant.
 - Immediate consequence: debt rises to $B_1 = 1$.

Central Question:

- How will this temporary tax cut impact future fiscal conditions?

Repayment in Year 2

- Government's budget constraint (year 2):

$$B_2 = (1 + r)B_1 + (G_2 - T_2) \quad (10)$$

- Condition for full debt repayment at year-end ($B_2 = 0$):
- Replacing B_1 by 1 and B_2 by 0 and transposing terms gives

$$T_2 - G_2 = (1 + r)B_1 = (1 + r)$$

Intuition:

- To eliminate debt incurred from a temporary tax cut, the government must generate a primary surplus exactly equal to debt plus interest.
- **How to achieve this surplus?**

A Fiscal Policy: Decreasing Tax in Year 1

- Two possible adjustments:
 1. Reduce government spending (G).
 2. Increase taxes (T).
- Assuming government spending remains unchanged:
 - Required tax increase in year 2 is $(1 + r)$.
- Thus, a 1-unit tax cut in year 1 necessitates a larger future tax increase of $(1 + r)$.

Key Insight:

- Temporary fiscal relief today implies larger fiscal burdens tomorrow.

Repayment in Year t

- If repayment is delayed until year t , and from year 2 to $t - 1$, primary deficit = 0:

$$B_{t-1} = (1 + r)^{t-2}$$

- **How?**
- From year 2 to year $t - 1$ the primary deficit is equal to zero; so, debt at the end of year 2 is:

$$B_2 = (1 + r)B_1 + 0 = (1 + r)1 = (1 + r)$$

- With the primary deficit still equal to zero during year 3, debt at the end of year 3 is

$$B_3 = (1 + r)B_2 + 0 = (1 + r)(1 + r)1 = (1 + r)^2$$

- **Debt growth without repayment:**
 - Even without new primary deficits, existing debt grows exponentially due to accumulating interest.

Repayment in Year t

- Year t budget constraint (debt fully repaid at end-year):

$$0 = (1 + r)B_{t-1} + (G_t - T_t)$$

- Substitute $B_{t-1} = (1 + r)^{t-2}$:

$$T_t - G_t = (1 + r)^{t-1}$$

Key Implications:

- The longer the delay or the higher the interest rate, the greater the eventual required surplus.
- Temporary relief becomes increasingly expensive over time.

Do Not Repay: Stabilize Debt

- Debt stabilization: Keeping debt constant at existing level from a certain period onwards.
- For stabilization starting at year 2 ($B_2 = B_1 = 1$):

$$1 = (1 + r)B_1 + (G_2 - T_2)$$

- Primary surplus required:

$$T_2 - G_2 = r$$

Intuition (?):

- To stabilize debt, the government must run a primary surplus exactly equal to interest payments every year.

Do Not Repay: Stabilize Debt

- Stabilizing rather than repaying debt implies:
 - Permanently higher debt due to past deficits.
 - Permanent increase in taxes (or decrease in spending) equal to annual interest payments.

Conclusions:

- Past deficits have lasting effects; stabilizing debt requires permanent fiscal adjustments.
- Temporary deficits can lead to permanently higher taxation or reduced public services.

Summary – Key Lessons on Debt Dynamics

- Debt dynamics depend critically on:
 - Real interest rate relative to economic growth.
 - Fiscal policy (primary deficit or surplus).

Key Takeaways:

- Short-term deficits create long-term fiscal burdens.
 - Delaying fiscal adjustments amplifies future costs.
 - Debt stabilization requires continuous fiscal discipline.
- Understanding these dynamics is essential for sustainable fiscal policymaking.

Costs of Persistent Deficits

Law of motion for (or, The Evolution of) Public Debt

- Fundamental debt equation:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

- Debt-to-GDP ratio evolution:

$$\frac{B_t}{Y_t} = \frac{(1 + r_t)B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

- Define debt-to-GDP ratio clearly as:

$$b_t \equiv \frac{B_t}{Y_t}$$

- Thus:

$$b_t = \frac{(1 + r_t)Y_{t-1}}{Y_t} b_{t-1} + \frac{G_t - T_t}{Y_t}$$

- Assume GDP growth rate g_Y , hence:

$$\frac{Y_t}{Y_{t-1}} = 1 + g_Y$$

Law of motion for (or, The Evolution of) Public Debt

- Debt-to-GDP ratio equation becomes:

$$b_t = \frac{1 + r_t}{1 + g_Y} b_{t-1} + \frac{G_t - T_t}{Y_t}$$

Insights on dynamics:

- If primary deficit $(G_t - T_t) = 0$, absolute debt grows at rate r .
- However, debt-to-GDP grows approximately at net rate $r - g_Y$ (exact gross factor: $\frac{1+r}{1+g_Y}$).

Important determinants of debt-to-GDP ratio growth:

- Higher real interest rate (r_t) accelerates debt accumulation.
- Lower GDP growth rate (g_Y) makes debt-to-GDP ratio increase faster.
- Higher initial debt ratio increases sensitivity to interest-growth differential.
- Larger primary deficits further drive up the ratio.

Maths

Useful approximation:

$$\frac{1+r}{1+g_Y} \approx 1+r-g_Y$$

Proof (Intuition):

- Consider:

$$(1+r-g_Y)(1+g_Y) = 1+r+rg_Y-g_Y^2$$

- If r and g_Y are small, products rg_Y and g_Y^2 become negligible:

$$(1+r-g_Y)(1+g_Y) \approx 1+r$$

- Dividing by $1+g_Y$ yields:

$$\frac{1+r}{1+g_Y} \approx 1+r-g_Y$$

Numerical Example:

- If $r = 0.05$, $g_Y = 0.03$, exact: 1.019; approximate: 1.02 (close approximation).

Sustainability of Public Debt

Long-term sustainability analysis:

- Assume zero primary deficits beyond time t_0 : $G_t = T_t$ for all $t \geq t_0$.
- Constant real interest rate $r_t = r$ for $t \geq t_0$.
- Debt-to-GDP ratio evolves as:

$$b_t = \frac{1+r}{1+g_Y} b_{t-1}$$

- Thus, explicitly:

$$b_t = \left(\frac{1+r}{1+g_Y} \right)^{t-t_0} b_{t_0}$$

$$b_t \approx (1+r-g_Y)^{t-t_0} b_{t_0} \quad (\text{first-order approximation})$$

Three critical cases:

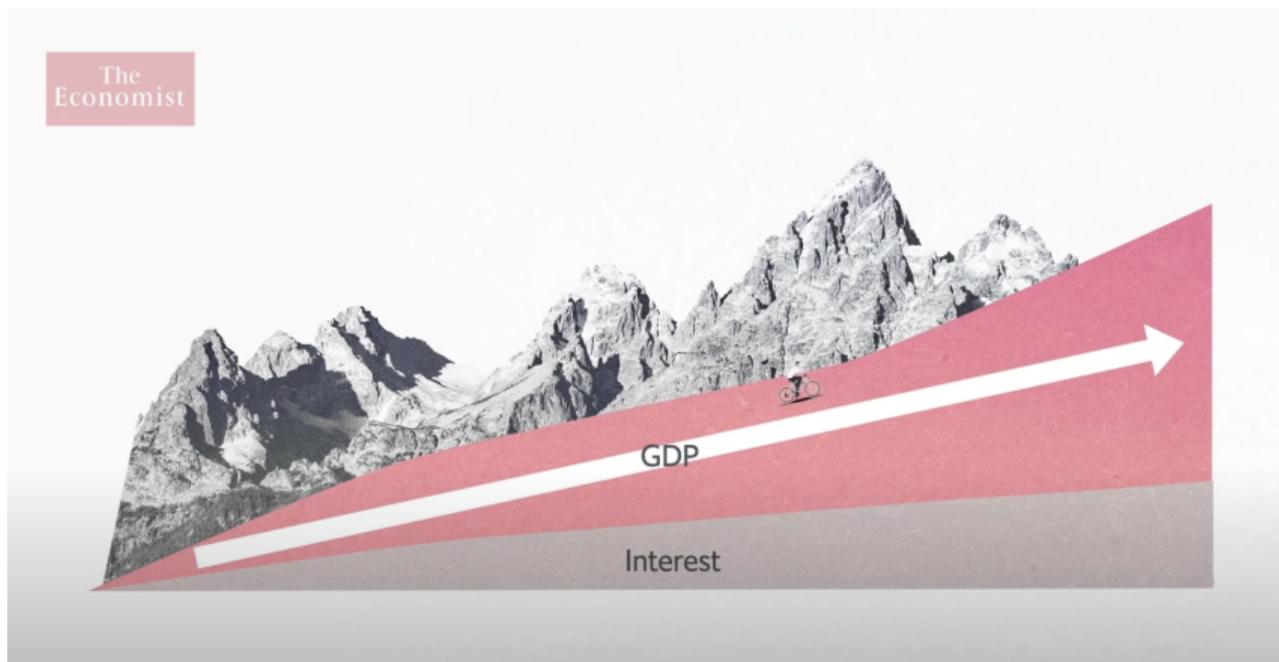
- If $r < g_Y$: with zero primary deficit, debt-to-GDP tends to fall over time.
- If $r = g_Y$: Debt-to-GDP remains constant (borderline sustainability).
- If $r > g_Y$: Debt-to-GDP ratio grows exponentially without bound, indicating unsustainability.

Crucial Insight: The sustainability of debt depends fundamentally on the interest-growth differential $(r - g_Y)$.

Some Questions: if $g_Y > r$ (Evidence 1)

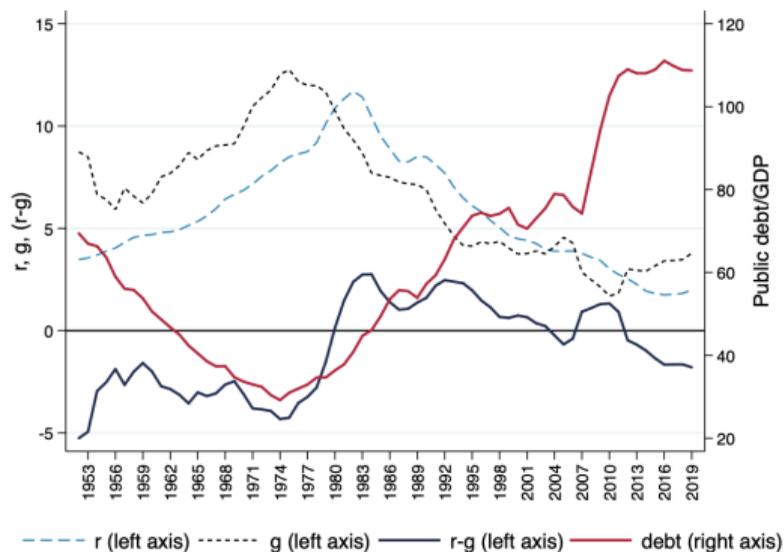


Some Questions: if $g_Y > r$ (Evidence 2)



Is public debt sustainable in the world?

Source: IMF Working Paper WP/20/137: *Public Debt and $r - g$ at Risk* by Weicheng Lian, Andrea F. Presbitero, and Ursula Wiriadinata



This figure plots the time series of the world's interest rate r , growth rate g , interest-growth differential $r - g$, and public debt-to-GDP ratio based on 17 advanced economies over the period 1950-2019. The world's values are GDP-weighted averages. All variables are 5-year moving averages. $r - g_t$ are computed as nominal local currency long-term rates r_t minus nominal local currency annual growth rates g_t .

Cost of Persistent Deficits

Persistent deficits have several significant economic costs:

- **Higher interest payments:**
 - Increased debt requires greater interest payments, reducing resources for productive public investments.
- **Reduced national saving and investment (crowding out):**
 - Government borrowing reduces funds available for private investments, potentially harming long-term growth.
- **Fiscal unsustainability and default risk:**
 - Unsustainable deficits lead to higher borrowing costs, risk sovereign debt crises, and erode investor confidence.
- **Increased future tax burden:**
 - Current deficits inevitably demand future tax increases or spending cuts, burdening future generations and distorting economic incentives.

Cost of unsustainable debt

Unsustainable debt occurs when $r > g_Y$:

$$b_t = \left(\frac{1+r}{1+g_Y} \right)^{t-t_0} b_{t_0}$$

Consequences of unsustainability:

- **Sovereign debt crisis:** Sudden loss of investor confidence and sharp rise in borrowing costs.
- **Severe fiscal adjustments:** Required austerity measures (spending cuts, higher taxes) leading to economic contractions.
- **Economic disruptions:**
 - Financial crises (banking instability)
 - Exchange rate crises (currency depreciation)
 - Twin crises (simultaneous financial and currency crises)

Strategic Debt Accumulation

Deficit Bias – Political Economy Overview

- Governments often run persistent deficits despite clear economic costs, a phenomenon known as **deficit bias**.
- Such deficits can arise from political frictions, not only from optimal tax-smoothing motives.
- The broad political-economy framework highlights two benchmark mechanisms:
 - **Strategic debt accumulation** (Tabellini and Alesina, 1990)
 - **Delayed fiscal stabilization** due to bargaining conflicts (Alesina and Drazen, 1991)

Strategic Debt Accumulation – Setup

- Two-period economy (periods 1 and 2).
- Government receives fixed endowment W each period.
- For simplicity in this block, assume zero real interest rate on debt ($r = 0$).
- Two types of public spending: goods M and N .
 - Example: Military vs. Healthcare
- Period-1 policymaker:
 - Chooses spending on M_1 , N_1 , and amount of debt B .
 - Budget constraint:

$$M_1 + N_1 = W + B$$

- Period-2 policymaker faces budget constraint:

$$M_2 + N_2 = W - B$$

- Borrowing ($B > 0$) increases current spending at future expense.
- Feasibility condition in this setup: $0 \leq B \leq W$.

Polarized Preferences – The Strategic Element

- Two types of policymakers with opposing preferences:
 - **Type-1:** Only values good M (utility $U(M)$).
 - **Type-2:** Only values good N (utility $U(N)$).
 - Standard assumptions on utility: $U' > 0, U'' < 0$.

- Type-1 policymaker in period 1 maximizes spending on good M :

$$M_1 = W + B, \quad N_1 = 0$$

- Period-2 policymaker uncertain:
 - Probability π : Type-1 continues spending on M .
 - Probability $1 - \pi$, Type-2 shifts spending entirely to good N .

Optimal Debt Choice and Strategic Behavior

- Type-1 policymaker maximizes expected utility by choosing debt B :

$$\max_{0 \leq B \leq W} U(W + B) + \pi U(W - B) + (1 - \pi)U(0)$$

- First-order condition for optimality:

$$\frac{U'(W + B)}{U'(W - B)} = \pi$$

- Analysis:

- If future policymaker is certainly Type-1 ($\pi = 1$), optimal debt $B = 0$.
- If future policymaker uncertain ($\pi < 1$), optimal debt is $B > 0$, strategically limiting future policymaker's choices.
- The lower π , the higher the incentive to accumulate debt.

Intuition behind Strategic Debt Accumulation

- Strategic debt arises because current policymakers prefer to allocate resources according to their priorities, anticipating potential misallocation by future policymakers.
- Debt is strategically used as a commitment mechanism to transfer resources from an uncertain future to the current period, aligning resource use with current policymaker preferences.

Economic Consequences:

- Inefficient intertemporal allocation of resources.
- Persistent deficits due to political polarization.
- Long-term fiscal burdens and suboptimal spending patterns.

Policy Implications:

- Political polarization can exacerbate deficit bias.
- Institutional arrangements (fiscal rules, independent councils) can mitigate strategic debt incentives.

Delayed Stabilisation

Delayed Stabilization – An Overview

Delayed stabilization: Persistence of large fiscal deficits despite known economic costs.

- Deficits cause economic distortions and risk sovereign debt crises.
- Timely fiscal reforms (tax increases or spending cuts) can mitigate these issues.
- Yet, reforms often delayed due to political conflicts and bargaining among interest groups.
- Historical example: Post-WWI hyperinflation prolonged by disagreement over taxing capital vs. labor.
- Modern debates: Disagreements over spending cuts vs. tax hikes.
- Alesina and Drazen (1991) model illustrates delays due to political bargaining.

Model Setup – Bargaining Framework

- Two groups: Capitalists and workers (both risk-neutral).
- Must agree on fiscal reform and distribution of fixed tax burden $T > 0$.
- Without agreement: severe fiscal crisis; both groups receive zero payoff.
- With agreement:
 - Workers earn pre-tax income W (assume $W > T$).
 - Capitalists earn uncertain pre-tax income R , uniformly distributed $[A, B]$, known only to capitalists.
- Tax agreement involves capitalists paying X of tax T :
 - Capitalists' payoff: $R - X$
 - Workers' payoff: $W - T + X$
- Feasible agreements beneficial for both groups if $0 \leq X \leq A$.

Bargaining and Probability of Acceptance

- Workers propose a tax contribution X ("take-it-or-leave-it" offer).
 - If capitalists accept, reform implemented; otherwise, crisis occurs.
- Capitalists accept if $R \geq X$.
- Probability of acceptance given proposal X :

$$P(X) = \begin{cases} 1 & \text{if } X \leq A \\ \frac{B-X}{B-A} & \text{if } A < X < B \\ 0 & \text{if } X \geq B \end{cases}$$

- Workers uncertain about R but aware of acceptance probability $P(X)$.

Workers' Optimal Proposal

- Workers' expected payoff:

$$V(X) = P(X)(W - T + X) = \begin{cases} W - T + X & \text{if } X \leq A \\ \frac{(B-X)(W-T+X)}{B-A} & \text{if } A < X < B \\ 0 & \text{if } X \geq B \end{cases}$$

- Workers have two strategic options:
 - Propose $X = A$ ensuring certain acceptance; payoff: $W - T + A$.
 - Propose $X \in (A, B)$, accepting uncertainty for potentially higher payoff.
- Optimal proposal depends on maximizing expected payoff $V(X)$.

Optimal Decision and Equilibrium Probability

- First-order condition for optimal X when $A < X < B$:

$$V'(X) = \frac{B - (W - T) - 2X}{B - A} = 0 \quad \Rightarrow \quad X = \frac{B - (W - T)}{2}$$

- This is the interior optimal solution—the point at which marginal gain from higher taxes equals the marginal loss from lower probability of acceptance.
- Workers choose:

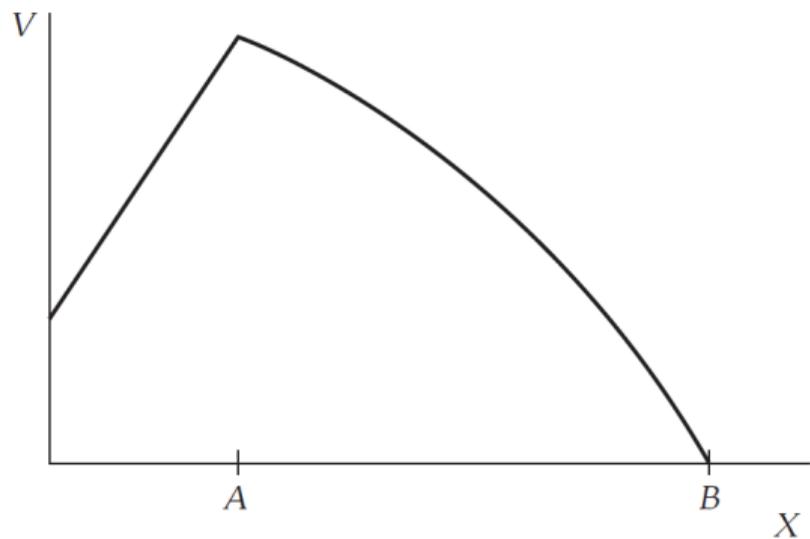
$$X^* = \begin{cases} A & \text{if } B - (W - T) - 2A \leq 0 \\ \frac{B - (W - T)}{2} & \text{if } B - (W - T) - 2A > 0 \end{cases}$$

- Equilibrium acceptance probability:

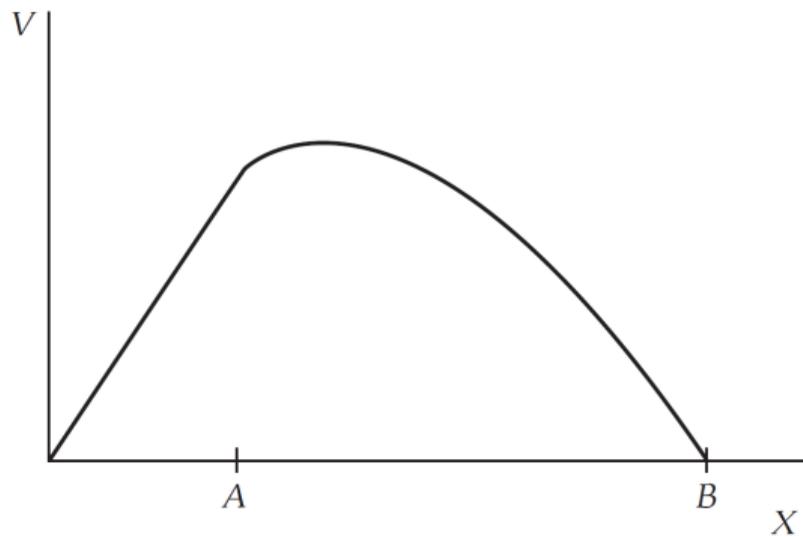
$$P(X^*) = \begin{cases} 1 & \text{if } B - (W - T) - 2A \leq 0 \\ \frac{B + (W - T)}{2(B - A)} & \text{if } B - (W - T) - 2A > 0 \end{cases}$$

- Higher uncertainty (larger $B - A$) reduces reform probability.

Examples: workers' expected payoff as a function of X



Workers pick $X^* = A$



Workers pick $X^* > A$

Implications of Delayed Stabilization Model

- Even mutually beneficial fiscal reforms can fail due to strategic bargaining.
- Workers may deliberately propose riskier terms seeking higher payoffs, increasing the likelihood of reform failure.
- Higher uncertainty in income distribution intensifies bargaining conflicts, delaying reforms.
- Countries with fragmented or weak governments (no clear decision-maker) are more prone to persistent deficits and delayed stabilization.
- Crisis situations (high costs of non-agreement) can sometimes incentivize rapid agreements, highlighting the complex role crises play in triggering necessary fiscal reforms.

Empirical Application: What Is Being Tested?

- Empirical question: do weak/fragmented governments run larger deficits, and are deficits more persistent under weak governments?

Roubini-Sachs style panel specifications:

$$D_{it} = a + bWEAK_{it} + c'X_{it} + e_{it}$$

$$D_{it} = a + b_1WEAK_{it} + b_2D_{i,t-1} + b_3(D_{i,t-1} \times WEAK_{it}) + c'X_{it} + e_{it}$$

- D_{it} : deficit-to-GDP outcome; $WEAK_{it}$: weak/fragmented government indicator.
- X_{it} : macro controls (growth, unemployment, external conditions).
- Interpretation of coefficients: b_1 captures level differences; b_3 tests whether political weakness amplifies deficit persistence.

Empirical Findings and Interpretation

- Baseline result: weaker governments are associated with larger deficits (in Roubini-Sachs, roughly about +1.2% of GDP for a one-step increase in weakness).
- Dynamic specification: deficits are persistent overall (b_2 is large), but the extra persistence under weak governments (b_3) is small and often statistically weak.
- So the strongest pattern is **higher deficit levels** under political fragmentation; evidence for much stronger **persistence** under weak governments is more limited.
- This is informative but not a clean causal design; case evidence and institutions still matter for interpretation.

Sovereign Debt Crises

Sovereign Default - Definition

→ **Narrow definition:** when the debtor violates the legal terms of the debt contract:

- Fail to pay interest or principal within the specified period.
- Breach some other contractual provision.
- This narrow definition overlooks situations in which the sovereign threatens to default and creditors respond by *voluntarily* revising the contract.

→ **Credit ratings agencies' definition:** when the sovereign breaks the debt contract or when the sovereign tenders a **distressed debt exchange** (i.e. an exchange offer of new debt with less favourable terms than the original debt).

- **Example:** Greece's debt restructuring did not trigger a narrow default: the government did not miss any payments and investors have not (successfully) alleged a technical breach. But the new terms offered by Greece were worse than those on the original debt, and ratings agencies conclude that a default had occurred.

Sovereign Debt Crises – An Overview

A government can't meet its debt obligations. This could happen either because it genuinely lacks sufficient funds (**insolvency**) or temporarily lacks access to sufficient liquidity (**illiquidity**).

Why might these crises occur suddenly?

- Crises often occur suddenly due to a rapid shift in investor sentiment.
- Even a small shift in economic fundamentals or investor beliefs can trigger a sudden refusal by investors to roll over existing debt.

Important questions addressed by the model:

1. Investor refusal to lend:
 - Investors refuse to purchase government bonds regardless of the interest offered due to a perceived high default risk, causing a liquidity-driven crisis.
2. Sudden crises from minor changes:
 - Small deteriorations in economic conditions (e.g., slightly higher debt or slightly lower expected revenue) can lead to large swings in investor sentiment. Thus, crises can happen quickly and unpredictably.

Model Setup – Rolling Over Government Debt

- Government has existing debt B maturing, with no immediate liquidity.
- To avoid default, the government must roll over debt by issuing new bonds to investors.
- Investors (risk-neutral) require minimum return $\bar{R} = 1 + \bar{r}$.
- Government offers interest rate $R = 1 + r$ (endogenous). Here R is a one-period gross return (distinct from $R(t)$ used earlier in discounting).
- Future tax revenue T is uncertain, described by cumulative distribution $F(T)$.
- If tax revenue $T \geq RB$, government repays debt.
- If $T < RB$, government defaults fully (all-or-nothing default).

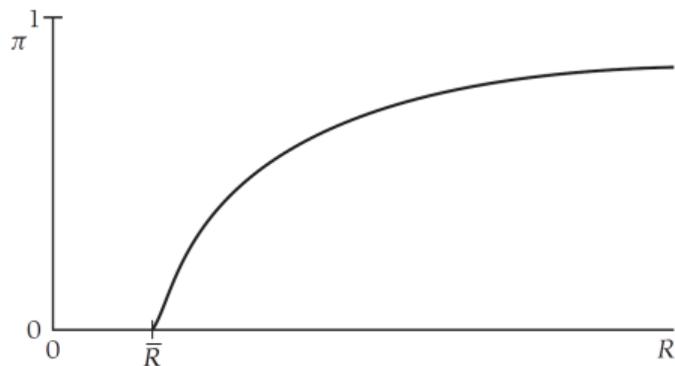
Investor Decision – Debt Demand Condition

- Investors' willingness to hold debt depends on the probability of default (π):

$$(1 - \pi)R = \bar{R}$$

- When no default risk ($\pi = 0$), $R = \bar{R}$.
- As $\pi \rightarrow 1$, investors require $R \rightarrow \infty$.
- Debt demand curve (downward-sloping relationship between default probability and interest rate):

$$\pi = 1 - \frac{\bar{R}}{R}$$

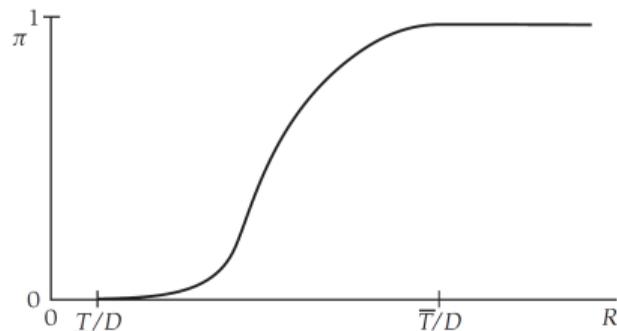


Probability of Default Condition

- Probability of default is the likelihood that future tax revenue T are insufficient to repay debt obligations RB :

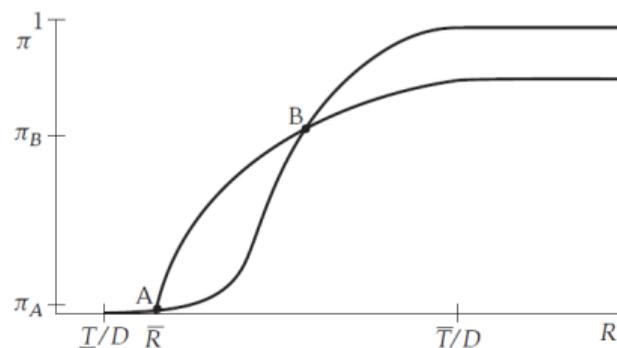
$$\pi = \Pr(T < RB) = F(RB)$$

- Typically, the distribution of tax revenue T is bell-shaped, yielding an S-shaped default probability curve:
 - At low interest rates, default probability is very low.
 - Probability sharply increases as R rises.
- Extreme outcomes:
 - $\pi = 0$ if RB is below minimum revenue (certain repayment).
 - $\pi = 1$ if RB exceeds maximum possible revenue (certain default).



Equilibrium – Multiple Equilibria

- Equilibrium defined by intersection of debt demand and default probability curves.
- Due to the curves' shapes, multiple equilibria possible:
 - Equilibrium A: Low default risk and interest rate close to safe rate \bar{R} .
 - Equilibrium B: High default risk and high interest rate.
- Additionally, there always exists a third equilibrium (certain default), where no investor is willing to buy government bonds at any finite interest rate.

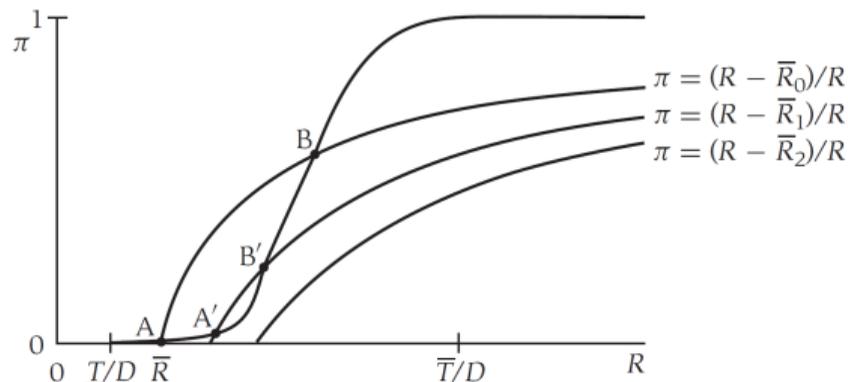


Stability of Equilibria and Investor Expectations

- Equilibrium stability determined by investors' expectations:
 - Equilibrium B (high risk) unstable: Small changes in expectations push outcomes toward extremes (either low-risk equilibrium or complete default).
 - Equilibrium A (low risk) and complete default equilibrium are stable.
- Self-fulfilling nature of crises:
 - Investor pessimism (expecting default) becomes self-fulfilling due to higher interest rates, increasing default risk.
 - Investor optimism maintains low default probabilities and interest rates.

Implications – Fundamentals and Crises

- Fundamentals strongly affect equilibrium outcomes:
 - Higher required safe return \bar{R} , increased debt B , or lower expected tax revenue shift equilibrium toward higher default risk.
- Small changes in fundamentals can trigger large shifts:
 - A modest rise in safe interest rate or debt levels can cause dramatic shifts from low-risk equilibrium to complete default.
 - Crises can thus arise suddenly and unexpectedly, even from minor economic changes.



Real-World Policy Implications

- Sovereign debt crises involve self-fulfilling dynamics:
 - Policy interventions that reduce investor uncertainty or improve credibility (e.g., clear fiscal commitments, credible monetary policies) can stabilize low-risk equilibria.
- National and international institutions (IMF, ECB, Central Banks) may act as lenders of last resort, stabilizing expectations.
- Preventive fiscal policy and maintaining sustainable debt levels critical to avoiding debt crises.

Summary and Conclusions

- Sovereign debt crises driven by interplay of economic fundamentals and investor expectations.
- Multiple equilibria highlight importance of investor beliefs:
 - Investor confidence reduces default likelihood.
 - Investor fear can cause self-fulfilling debt crises.
- Sudden shifts to default equilibrium possible with minor fundamental changes.
- Robust fiscal frameworks and credible economic policy are key to crisis prevention.