Unemployment
- WEEK SIX -

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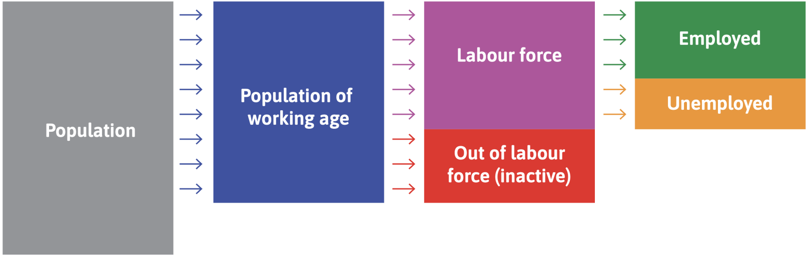
Introduction and Terminology

Unemployment

According to the standardised definition of *the International Labour Organization* (**ILO**):

- **Employment** is the number of people who have a job.
- **Unemployment** is the number of people who do not have a job but are looking for one.
  - an unemployed person is a person aged 15 or over:
  - without a job during a given week;
  - available to start a job within the next two weeks;
  - actively having sought employment at some time during the past four weeks or having already found a job that starts within the next three months.



Terminology

- N is the working-age population,
- Q is the labour force (employed + unemployed)
- U is the number of unemployed,

$$Unemployment\ Rate = \frac{U}{Q} \tag{1}$$

$$Participation\ Rate = \frac{Q}{N} \tag{2}$$

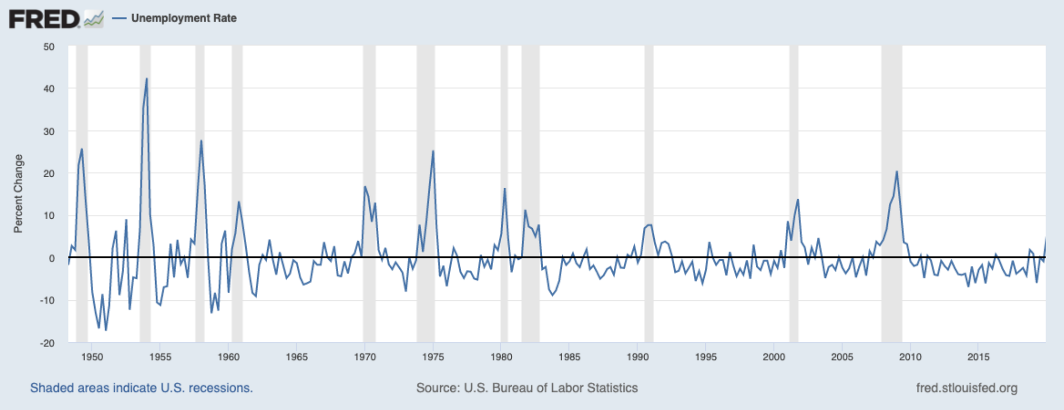
- the employment/population ratio

$$\frac{Employment}{Population} = \frac{Q - U}{N} \tag{3}$$

How to measure?

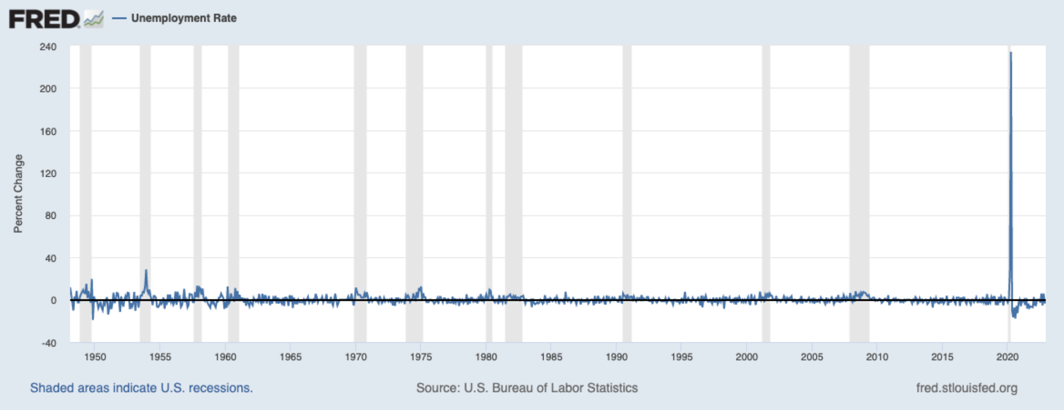
- **Difficult!**
- Many countries rely on large surveys of households to compute the unemployment rate.
- The U.S. Current Population Survey (C P S) relies on interviews of 60,000 households every month.
- The two measures of unemployment used in the **UK** are the **Claimant count** and **the Labour Force Survey**. Between the two, the Labour Force Survey is considered a more accurate measure of unemployment.
- A person is unemployed if he or she does not have a job and has been looking for a job in the last four weeks.
- Those who do not have a job and are not looking for one are counted as **not in the labour force.**
- **Discouraged workers** are those persons who give up looking for a job and so no longer count as unemployed.
- **The participation rate** is the ratio of the labour force to the total population of working age.
- Because of discouraged workers, a higher unemployment rate is typically associated with a lower participation rate.

US Unemployment Rate - 1950-2019



SOURCE. FRED. Series UNRATE. .

US Unemployment Rate - 1950-2022



Source. FRED. Series UNRATE. .

Why it matters?

Why Do Economists Care about Unemployment?

1. Because of its direct effect on the welfare of the unemployed, especially those remaining unemployed for long periods of time.

2. It is a signal that the economy is not using its human resources efficiently.

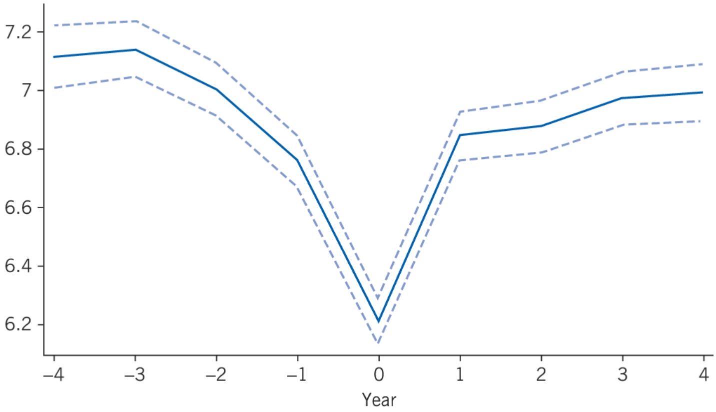
   When unemployment is high:

   * Many workers who want to work do not find jobs; the economy is clearly not using its human resources efficiently.

   What about when unemployment is low?

   * An economy in which unemployment is very low may be overusing its resources and run into labour shortages (particularly for skilled workers).

Unemployment and Happiness



SOURCE. Rainer Winkelmann, IZA, 2014. .

To give you a sense of scale, other studies suggest that this decrease in happiness is close to the decrease triggered by a divorce or a separation.

Unemployment and Happiness

SOURCE. Rainer Winkelmann, IZA, 2014. .

Unemployment and Happiness

A photo from the Great Depression in the 1930s
US unemployment reached 25%

Two Main Issues in the Labour Market

There are two main groups of issues we are interested in:

1. The determinants and consequences of **average unemployment**, e.g.
   * Why does the labour market not clear, i.e. why do wages not fall in the face of significant unemployment?
   * Why does unemployment vary across countries and over time?
   * What are the welfare consequences of normal unemployment?
2. **Cyclical behaviour** of the labour market, e.g.
   * Why does employment fluctuate a lot more than real wage?
   * Why do firms lay off workers in downturns rather than rely on work-sharing sharing arrangements?

Failure of Walrasian models of the labour market

- Recall that in the RBC and New Keynesian models we have considered so far the labour markets are Walrasian:
  - Wage always clears the market.
  - Individuals are always on their optimal labour supply.
  - There is no unemployment, only choices to consume more leisure.
- Fluctuations in employment in these models reflect willingness of people to substitute labour and leisure across periods.
  - Yet empirical studies find little evidence of significant intertemporal substitution, and point to inelastic individual labour supply.
  - This mechanism predicts counterfactually large fluctuations in the real wage, but much less volatility in employment than in the data.
- **Key question:** Why do shifts in labour demand lead to large movements in employment, and only small changes in the real wage?

The game plan

- We will consider three highly influential models of the labour market:
  1. **Efficiency wage theory** (warm-up)
  2. **Shapiro-Stiglitz model,** which formally explores the deeper reasons for efficiency wages.
  3. **Search and matching model.**
- The first two are examples of the **traditional approach** of modelling the labour market within the standard supply and demand framework.
- The last one is an example of the **modern approach.**
  - ⇒ Focus on the heterogeneity among workers and jobs, and the costly process of job search and recruitment.

Efficiency Wages

Efficiency wage theories: overview

- The key idea of efficiency wage theories is that there are benefits of paying higher wages to employees.
- Among suggested reasons, the following received the most attention:
  1. Better **nourishment**, and thus productivity.
  2. **Incentive to exert high effort** when firms cannot monitor workers perfectly, as in the Shapiro-Stiglitz (1984) model – later today.
  3. Higher wages can **attract workers of higher ability**.
  4. **The fair wage-effort hypothesis** due to Akerlof and Yellen (1990): high wage can build loyalty and hence induce effort – homework.
     $\Rightarrow$ extensive evidence that workers' effort is affected by such feelings as anger, jealousy, and gratitude.
- We begin with a simple efficiency wage model due to Solow (1979).

Setup

- No capital for simplicity, labour is the only factor of production.
- There is a large number N of firms and a representative firms maximizes profits:

$$\pi = Y - wL, \tag{4}$$

- where Y is the firm's output, w is the wage that it pays, and L is the amount of labor it hires.
- Output depends on the number of workers and on their effort; $Y = F(eL)$.
- Thus the representative firm's output:

$$\pi = F(eL) - wL, \quad F'(\bullet) > 0,\ F''(\bullet) < 0, \tag{5}$$

where e is workers' **effort**, so eL is **effective labour**.

Setup

- Assume effort e is an increasing function of the wage:

$$e = e(w), \quad e'(\bullet) > 0 \tag{6}$$

- For now, we are interested in the implications, and not precise reasons.
- There are $\bar{L}$ workers, each supplying 1 unit of labour *inelastically*

  $\Rightarrow$ i.e prepared to work at any wage.

The firm's problem

- A representative firm solves

$$\max_{L,w} F(e(w)L) - wL. \tag{7}$$

- There can be two cases:
  1. There are unemployed workers and the firm can choose its wage freely.
  2. There is zero unemployment, so the firm must pay at least the wage paid by other firms to attract any workers.

The firm's problem

- In either case, the firm is free to choose its employment level.
  FOC w.r.t. L yields:

$$F'(e(w)L) = \frac{w}{e(w)}. \tag{8}$$

- I.e. marginal product of *effective* labour equals its unit cost, $w/e(w)$.
  Note: When a firm hires a worker, it gets $e(w)$ units of effective labour.

The efficiency wage

- When the firm is unconstrained and sets wage freely, taking FOC w.r.t. w from (7) yields

$$F'(e(w)L)Le'(w) - L = 0. \tag{9}$$

- Substitute for $F'(e(w)L)$ from (8) and rearrange to get:

$$\frac{we'(w)}{e(w)} = 1 \tag{10}$$

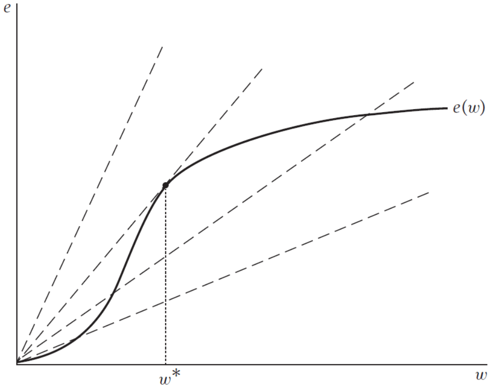
- I.e. at the optimum, elasticity of effort w.r.t. wage is 1.

Intuition:

  * The firm wants to hire effective labour, eL, as cheaply as possible.
  * (10) defines the **efficiency wage** that **minimizes the unit cost** of effective labour, i.e. solves $\min_w w/e(w)$ – you can check it.
  * Put differently, optimal w **maximizes effort per dollar spent**, $e(w)/w$

The efficiency wage

$$\frac{we'(w)}{e(w)} = 1 \qquad (11)$$



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Equilibrium

- Let L^* and w^* denote the values that satisfy conditions (8) and (10).
- Since all firms are identical, the total labour demand at w^* is NL^*.
- If $NL^* < \bar{L}$, then there is **positive unemployment** in equilibrium:
  - Firms are free to set wages, so the equilibrium wage is simply w^*.
  - At this wage, employment is indeed given by the labour demand, NL^*.
  - $\bar{L} - NL^*$ workers are unemployed.
- If $NL^* > \bar{L}$, there is **full employment** in equilibrium:
  - At w^*, labour demand would exceed supply,
  - so the wage is bid up above w^* in equilibrium until $NL(w) = \bar{L}$.
  - Firms are constrained, and unable to reduce the wage to w^*.

Implications

1. The model implies the possibility of **involuntary unemployment:**
   * When $NL^* < \bar{L}$ and $w = w^*$, there are workers who want to work at the prevailing wage, yet cannot find employment.
   * The wage does not fall to equilibrate labour supply and demand, since it reflects efficiency considerations of maximizing workers' effort.
2. The model also sheds light on the labour market dynamics over the business cycle:
   * There is no reason for the firms to adjust real wages in response to, say, a negative demand or productivity shock.
   * The model thus predicts that shifts in labour demand will lead to large movements in employment, with little changes in the wages.

Case study: Henry Ford and efficiency wages

- In 1914, Henry Ford instituted a $5 a day minimum wage for his workers.
- This was double the going wages at the time!
- Ford himself: “There was… no charity in any way involved. … The payment of $5 a day for an eight hour day was one of the finest cost cutting moves we ever made”


'GOLD RUSH' IS STARTED BY FORD'S $5 OFFER

Thousands of Men Seek Employment in Detroit Factory.

Will Distribute $10,000,000 in Semi-Monthly Bonuses.

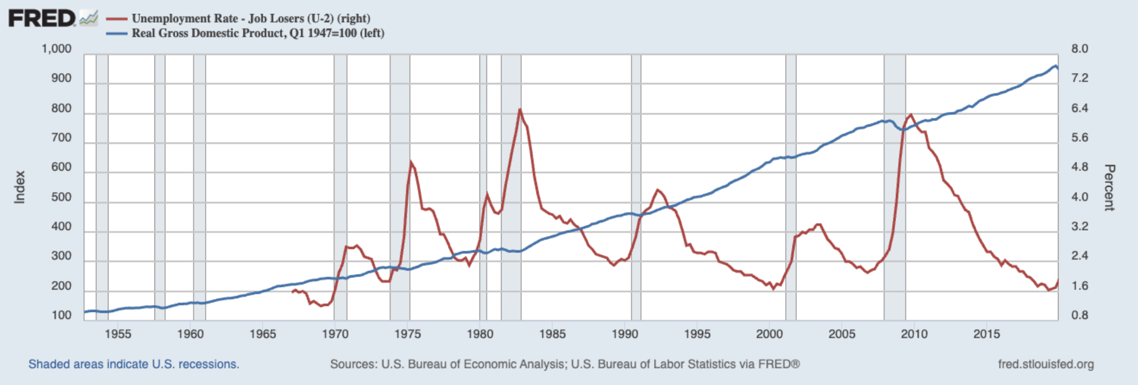
No Employe to Receive Less Than Five Dollars a Day.

(TIMES-STAR SPECIAL DISPATCH.)
DETROIT, Mich., January 7.—Henry Ford in an interview to-

A big BUT

- Unchanged Real Wage Over Time
  - In the model, as labor demand grows, the real wage remains constant for a long stretch.
  - Eventually, as unemployment falls to zero, any further increase in labor demand must raise the real wage.
- Mismatch with Observed Unemployment Trends
  - In reality, unemployment does not trend steadily toward zero.
  - The model's prediction of a declining unemployment rate over time clashes with long-run data, which shows no clear trend in unemployment.

Unemployment and GDP



Empirical Puzzle

- Short-Run vs. Long-Run Behavior
  - Short run: Labor-demand shifts primarily affect employment rather than the real wage.
  - Long run: Those same shifts should eventually move the real wage more than employment, but we do not observe that pattern.
- Unanswered Questions
  - Why do real wages not remain constant as demand grows over long periods?
  - How to reconcile no clear trend in unemployment with a model implying a downward trend?
  - The efficiency wage framework alone does not resolve these issues, indicating a need for additional mechanisms or explanations.

A More General Efficiency-Wage Framework

Why Extend the Basic Model?

- In the simplest efficiency-wage model, effort depends only on a single firm's wage.
- But in reality, workers also compare that firm's wage to what other firms pay, and they weigh the risk of being fired when unemployment is high or low.

- The basic efficiency-wage idea now includes:
  1. The wage the firm itself pays, w.
  2. The wage paid by other firms, w_a.
  3. The unemployment rate, u.

New Effort Function

$$e = e(w, w_a, u)$$

A More General Efficiency-Wage Framework

Effort Function

$$e = e(w, w_a, u),$$

subject to

$$\frac{\partial e}{\partial w} > 0, \quad \frac{\partial e}{\partial w_a} < 0, \quad \frac{\partial e}{\partial u} > 0.$$

Intuition:

1. Higher own wage (w) raises a worker's effort.
2. Higher outside wage (w_a) lowers effort (because alternative jobs look better).
3. Higher unemployment (u) raises effort (fear of job loss).

The Representative Firm's Problem

Production Side: Let the firm's production be $F(e(w, w_a, u)\, L)$

Profit Maximization: Choose w to balance wage costs against effort gains.
When the firm is price-taking in both product and labor markets, the key first-order conditions can be rearranged to:

$$F'(e(w, w_a, u)\, L) \;=\; \frac{w}{e(w, w_a, u)},$$

$$w\, \frac{\partial e / \partial w}{e(w, w_a, u)} \;=\; 1.$$

Interpretation:

- The marginal product of effective labor, F', must equal the ratio of the wage to effort.
- The elasticity of effort w.r.t. the firm's own wage is exactly 1 in equilibrium.

A Specific Example (Summers, 1988)

$$e = \begin{cases} \left(\dfrac{w - x}{x}\right)^{\beta} & \text{if } w > x, \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

For detailed explanation go to https://macroeconomics.info/.

where $x = (1 - bu)w_a$

Parameters:

- $0 < \beta < 1$.
- $b > 0$ measures how strongly unemployment, u, affects a worker's perceived outside options.

Mechanics:

- If $w \leq x$, workers exert no effort.
- For $w > x$, effort increases less than proportionally with $(w - x)$

Solving for Equilibrium Wages and Unemployment

1. Taking FOC (12) w.r.t. w yields:

$$\beta \frac{w}{[(w-x)/x]^{\beta}} \left(\frac{w-x}{x}\right)^{\beta-1} \frac{1}{x} = 1.$$

2. Equilibrium Wage Solving yields:

$$w = \frac{x}{1-\beta} = \frac{1-b\,u}{1-\beta}\, w_a.$$

3. Consistency (Firm Chooses the Prevailing Wage) Imposing $w = w_a$ yields:

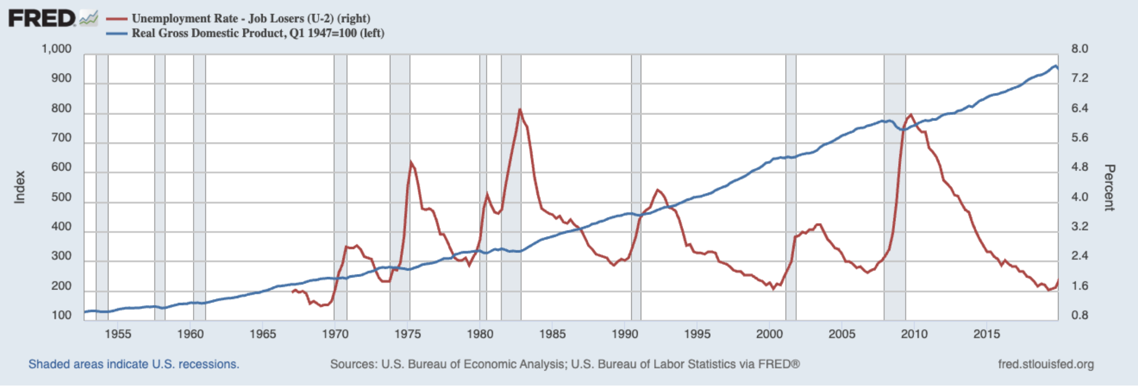
$$(1-\beta)\, w_a = (1-b\,u)\, w_a \implies u = \frac{\beta}{b}. \tag{13}$$

- **Interpretation**: The equilibrium unemployment rate, $\boldsymbol{u_{EQ}}$, depends **only** on β and $\boldsymbol{b}$ **and is independent of the long-run growth rate**.

Unemployment and GDP

Unemployment Rate is independent of the long-run growth rate

$$u = \frac{\beta}{b}$$



Shapiro-Stiglitz Model

Shapiro-Stiglitz (1984) model: an overview

- So far we have simply been assuming that workers' effort is an increasing function of the wage in (6).
- In the highly influential paper, Shapiro and Stiglitz (1984) provide a microeconomic rationale for this assumption.
- **Idea:** if firms have a limited ability to monitor their workers, they are forced to provide them with enough incentives to exert high effort.
- In the model such incentive arises from the risk of being fired and loosing a well-paid job if not working hard (shirking).
- Not only does the model provide a logical justification for the efficiency wage, but it also does it in a spectacularly elegant fashion.

Assumptions of the Model

- The economy has:
  - A large number of workers $\bar{L}$ and firms N.
  - Workers maximize **lifetime utility**:

$$U = \int_0^\infty e^{-\rho t} u_t \, dt, \quad \rho > 0 \tag{14}$$

  where u_t is the instantaneous utility at time t and ρ is the discount rate.
- Instantaneous utility:

$$u_t = \begin{cases} w_t - e_t, & \text{if employed, effort exerted} \\ 0, & \text{if unemployed} \end{cases} \tag{15}$$

- Effort e_t has two levels: $e_t = 0$ (shirking) or $e_t = \bar{e} > 0$ (effort).

Worker States

$$u_t = \begin{cases} w_t - e_t, & \text{if employed, effort exerted} \\ 0, & \text{if unemployed} \end{cases}$$

At any moment in time, a worker can be in one of three states:

- Employed and exerting effort $\boldsymbol{E}$ and getting utility $\boldsymbol{w_t - \bar{e}}$
- Employed and shirking $\boldsymbol{S}$ and getting utility $\boldsymbol{w_t}$
- Unemployed $\boldsymbol{U}$ and getting utility $\boldsymbol{0}$

Transitions between states follow simple Poisson processes.

Job Ends: Exogenous Reasons - b - From E to U

- Jobs end randomly (exogenously) at a rate b and $b > 0$.
- The probability of the job surviving at a some later time t is:

$$P(t) = e^{-b(t-t_0)} \tag{16}$$

For detailed explanation go to https://macroeconomics.info/.

- The equation (16) states that the probability of the job surviving decays exponentially, with faster decay for higher b.
- More importantly, (16) implies that $P(t+\tau)/P(t) = e^{-b\tau}$, which is the probability that the worker is still employed time τ later.
- Thus, the job surviving is independent of t, the length of employment.
- Thus, the risk of job loss is memoryless, or job breakups follow a Poisson process.

Job Ends: Shirking Detection - q - From S to U

- q is the probability per unit of time that a shirker is detected.
- q is exogenous and independent of job separations b.
- Firms detect shirkers and fire them.
- The probability that a shirker is still employed time τ later is $e^{-q\tau} \times e^{-b\tau}$ the probability that the job survives time τ later.
- Combined probability of survival and no shirking detection:

$$P(t) = e^{-b(\tau)} \cdot e^{-q(\tau)} = e^{-(b+q)(\tau)} \tag{17}$$

Can $q \rightarrow \infty$?

- Note that q measures how frequently shirkers are monitored and detected.
- If the firm uses highly effective monitoring technology q can become arbitrarily large.
- The probability of not being detected within τ is $e^{-q\tau}$
- If $q \rightarrow \infty$, $e^{-q\tau} \rightarrow 0$, so $P_{detected\ within\ \tau} = 1 - e^{-q\tau} \rightarrow 1$

Job Finding: a - From U to E (and/or to S)

- **Definition**: Probability per unit time an unemployed worker finds a job.
  - A worker is unemployed at time t, the probability that they are employed at time $t + dt$ is $a \cdot dt$.
  - a is determined endogenously by the labor market conditions.
- **Steady-State Condition**:

$$b \cdot E = a \cdot U \tag{18}$$

- **Expression for a**:

$$a = \frac{NLb}{\bar{L} - NL} \tag{19}$$

- **Economic Implications**:
  - Reflects labor market tightness. $\uparrow a \Rightarrow$ jobs are easier to find.
  - Influences worker incentives to exert effort. $\uparrow a \Rightarrow \downarrow e$
  - Guides policy interventions to reduce unemployment.

Firm's Profit Function

$$\pi(t) = F\left(\bar{e}L(t)\right) - w(t)\left[L(t) + S(t)\right] \tag{20}$$

where $F'(\bullet) > 0$ and $F''(\bullet) < 0$.

- $F\left(\bar{e}L(t)\right)$: Total output, where $\bar{e}$ is the effort level and $L(t)$ is the number of workers exerting effort.
- $w(t)[L(t) + S(t)]$: Total wage cost, where all employed workers are paid.

Firm's Objective:

- Maximize instantaneous profits $\pi(t)$ by setting:
  - Wage w high enough to deter shirking.
  - Employment $L(t)$ at the profit-maximizing level.
- Higher wages reduce shirking ($S(t) \to 0$) but increase wage costs.

Final Assumption and Full Employment

$$\bar{e}F'\left(\frac{\bar{e}L}{N}\right) > \bar{e}, \quad \text{or} \quad F'(\bar{e}L/N) > 1$$

- $\bar{e}F'(x)$: Marginal product of labor times effort.
- $\bar{e}$: Cost of exerting effort.
- Condition: Marginal product exceeds effort cost, ensuring full employment.

Economic Insight:

- Without shirking, firms hire until the marginal product of labor equals the cost of hiring.
- Imperfect monitoring creates equilibrium unemployment because firms pay efficiency wages to deter shirking.
- Higher wages reduce shirking but lead to fewer hires, creating involuntary unemployment.

Implications Intuitive Explanation:

- Marginal product of labor exceeds effort cost.
- Ensures full employment without monitoring issues.
- Hiring more workers is beneficial if output exceeds effort cost.
- Imperfect monitoring leads to unemployment despite this condition.

Dynamic Programming

Key Idea: Use value functions to summarize the future in dynamic programming.

State Values:

- Let V_i denote the **value of being in state** i.
- States include:
  - E: Employment
  - U: Unemployment
  - S: Shirking
- V_i represents the **expected discounted lifetime utility** from the present moment forward of a worker in state i.

Why Constant?

- Transitions among states are **Poisson processes**.
- V_i is independent of how long a worker has been in a state (steady state assumption).

Deriving V_E - Mathematically - 1/3

- In continuous time, time is divided into infinitesimally small intervals of length, Δt
- The Shapiro-Stiglitz model provides a framework to:
  - Evaluate the trade-offs between wages, effort, and unemployment risk.
  - Predict worker behavior (e.g., shirking).
- Let $V_E(\Delta t)$ and $V_U(\Delta t)$ denote the value of being employed and unemployed as of the beginning of an interval.
- Two components:
  - Utility from earning wages during Δt: $w - \bar{e}$
  - Future utility after Δt, accounting for:
    - Probability of remaining employed ($e^{-b\Delta t}$).
    - Probability of becoming unemployed ($1 - e^{-b\Delta t}$).

Note: When we let Δt approach zero, the constraint that a worker who loses his or her job during an interval cannot find a new job during the remainder of that interval becomes irrelevant. Thus $V_E(\Delta t)$ will approach V_E. Similarly, $V_U(\Delta t)$ will approach V_U.

Deriving V_E - Mathematically - 2/3

$$V_E(\Delta t) = \int_{t=0}^{\Delta t} e^{-(\rho+b)t}(w - \bar{e})\, dt + e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right] \tag{21}$$

For detailed explanation go to `https://macroeconomics.info/`.

Equation (21) has two parts:

1. $$\int_{t=0}^{\Delta t} e^{-(\rho+b)t}(w - \bar{e})\, dt$$

   represents the flow of utility from being employed during the interval $[0, \Delta t]$.

2. $$e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right]$$

   represents the expected utility from being employed or unemployed after Δt.

Deriving V_E - Mathematically - 3/3

- If we compute the integral in equation (21) we get:

$$V_E(\Delta t) = \frac{1}{\rho + b}\left(1 - e^{-(\rho+b)\Delta t}\right)(w - \bar{e}) + e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right] \tag{22}$$

- Solving this equation for $V_E(\Delta t)$ gives

$$V_E(\Delta t) = \frac{1}{\rho + b}(w - \bar{e}) + \frac{1}{1 - e^{-(\rho+b)\Delta t}}e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right]. \tag{23}$$

- Remember that $V_E = \lim_{\Delta t \to 0} V_E(\Delta t)$ (Similarly for V_U).
- To find this, apply L'Hôpital's rule to (23) then we get:

$$V_E = \frac{1}{\rho + b}(w - \bar{e}) + \frac{b}{\rho + b}V_U. \tag{24}$$

Deriving V_E - Intuitively - 1/3

- Think of employment as an "asset" that pays a stream of dividends over time:
  - **Dividends**: While employed, the worker earns utility per unit of time given by $\boldsymbol{w - \bar{e}}$.
  - When unemployed, no dividends are received.
- V_E is then the fair price of such an asset, which represents the **expected present value of all future dividends** a worker receives while employed, discounted at a required rate of return $\boldsymbol{\rho}$.
- The expected return must thus be $\boldsymbol{\rho V_E}$ per unit time.
- Lastly, there is a probability $\boldsymbol{b}$ per unit time of a 'capital loss' $\boldsymbol{(V_E - V_U)}$ if the worker gets unemployed, i.e. his 'asset' loses value.

$$\rho V_E = (w - \bar{e}) - b(V_E - V_U) \tag{25}$$

For detailed explanation go to https://macroeconomics.info/.

which is the same as equation (24).

Deriving V_E - Intuition Summary - 2/3

Key Insights:

- Employment is analogous to an asset that provides:
  - **Dividends:** $(w - \bar{e})$
  - **Capital Loss:** $b(V_E - V_U)$
  - **The required return on the asset** ρV_E
- Equation balances:

$$\rho V_E = (w - \bar{e}) - b(V_E - V_U)$$

- Rearranging gives the lifetime utility:

$$V_E = \frac{1}{\rho + b}(w - \bar{e}) + \frac{b}{\rho + b} V_U$$

Takeaway: Employment value reflects flow utility, transition probabilities, and discounting.

Deriving V_S and V_U - Intuitively - 3/3

- We can extend the asset analogy to the other two states.
- When a worker is shirking, the 'dividend' is w per unit time, but the rate at which he looses the job is also higher at $b+q$. Thus:

$$\rho V_S = w - (b+q)(V_S - V_U). \tag{26}$$

- When the worker is unemployed, he receives no 'dividend', but he gets a job ('capital gain') at the rate a, so

$$\rho V_U = a(V_E - V_U). \tag{27}$$

- We assumed above that if an unemployed worker gets a job, he exerts effort, as will indeed be the case in equilibrium.

Firms' problem

- A representative firm's profit per unit time is given (as in equation (20)) by:

$$\pi = F\left(1-\alpha\right)L - w\left[L+S\right], \quad F'(\bullet) > 0, \quad F''(\bullet) < 0. \tag{28}$$

- $L,\ S$ are numbers of workers who exert effort and shirk, respectively.
- The problem facing the firm is to incentivise employees not to shirk:
  - It must pay enough that $V_E \geq V_S$, otherwise workers prefer shirking.
  - The optimizing firm will not overpay more then necessary and will choose w so the incentive constraints is just satisfied, i.e. $V_E = V_S$.
- Use $V_S = V_E$ in (26), and then subtract (25) and rearrange to get:

$$V_E - V_U = \frac{\bar{e}}{q} > 0. \tag{29}$$

- The workers thus must *strictly* prefer employment to unemployment.

  $\Rightarrow$ to induce effort, firms pay a premium over&above the cost of effort $\bar{e}$.

Wage level that induces effort

- Subtract ρV_U in (27) from ρV_E in (25) to get:

$$\rho(V_E - V_U) = (w - \bar{e}) - (a + b)(V_E - V_U) \tag{30}$$

- Substitute $\frac{\bar{e}}{q}$ for $V_E - V_U$ from incentive cond. (29) and solve for w:

$$\boldsymbol{w = \bar{e} + (a + b + \rho)\frac{\bar{e}}{q}.} \tag{31}$$

Wage level that induces effort

$$w = \bar{e} + (a + b + \rho)\frac{\bar{e}}{q}.$$

- This is the wage level needed to induce effort, which
  - exceeds the cost of effort $\bar{e}$ by a positive amount.
  - increases in the cost of effort $\bar{e}$, the ease of finding jobs a, the rate of job breakup b, and the discount rate ρ.
  - decreases in the rate at which shirkers are detected q.
- The firms will pay this wage so there is no shirking in equilibrium.

The aggregate no-shirking condition (NSC)

- Since the economy is in a steady state, the number of unemployed workers is constant, so flows into and out of unemployment balance:
  - NLb workers become unemployed per unit time (where L is employment per firm and so NL is aggregate employment).
  - $a(\bar{L} - NL)$ unemployed workers find jobs per unit time.
  - Equating the two, we get the equilibrium job-finding rate as in equation (19):

$$a = \frac{NLb}{\bar{L} - NL}. \tag{32}$$

The no-shirking condition (NSC) - Reminder -

- Substitute this into (31) to get the **no-shirking condition (NSC):**

$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b \right) \frac{\bar{e}}{q}. \tag{33}$$

- No-shirking wage is an increasing function of aggregate employment:
  - When $NL \uparrow$, it is easier for unemployed workers to find jobs, see (19).
  - The cost of being fired falls, so wage has to rise to prevent shirking.

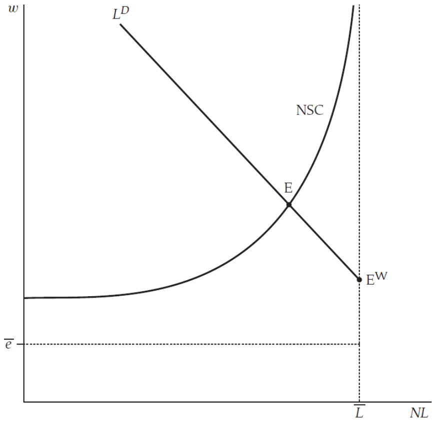
Equilibrium - 1/2 -

- The FOC of a firm's profit function (28) w.r.t. L yields

$$\bar{e}F'(\bar{e}L^*) = w, \tag{34}$$

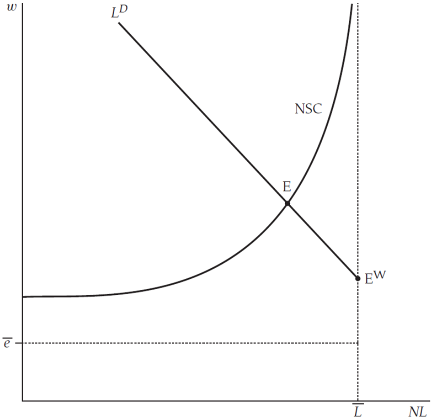
  so firms hire workers until marginal product of labour equals the wage.

- This implies downward sloping aggregate labour demand $L^D = NL^*$.
- In the absence of any monitoring issues, Walrasian equilibrium would occur at point E^W where L^D crosses the inelastic labour supply $\bar{L}$.
  - I.e. there would be full employment in equilibrium (assuming that the marginal product of labour at full employment exceeds cost of effort $\bar{e}$).



Equilibrium - 2/2 -

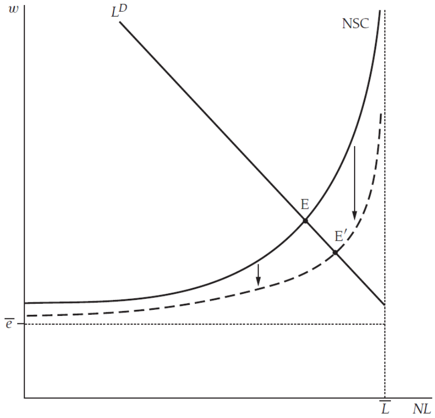
- However, with imperfect monitoring and possible shirking, equilibrium occurs at the intersection E of L^D and the no-shirking condition.
  - Wage is above the Walrasian level and there is positive unemployment.
  - Unemployed workers strictly prefer to be employed and exert effort,
  - but the wage does not fall, because then workers would start shirking.



The effect of a rise in q

An increase in the shirking detection rate q shifts the NSC down.

- The equilibrium wage falls and employment rises.
- Intuition: monitoring becomes better, so there is less need to incentivise workers by high wage.
- As $q \to \infty$, the economy approached the Walrasian equilibrium.

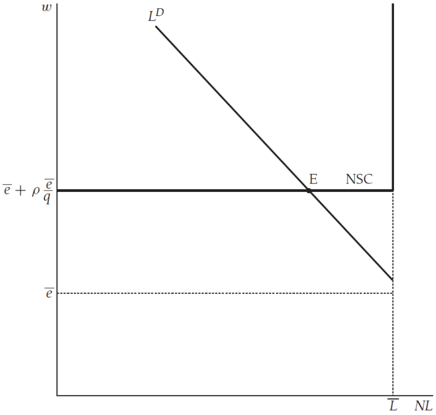


With turnover: $b = 0$

If the job separation rate b falls to 0, there is no turnover, and unemployed workers are never hired.

$\Rightarrow$ The no-shirking wage in this case is simply $\bar{e} + \rho\bar{e}/q$, see (33)

- i.e. the NSC becomes flat and independent from employment.
- Intuitively, workers now only consider the cost of effort and the risk of permanently losing employment when contemplating shirking.



Homework invites you to conduct some additional exercises.

What have we achieved?

- Like any efficiency wage theory, Shapiro-Stiglitz model implies:
  1. There is involuntary unemployment.
  2. Although individual labour supply is inelastic when $w > \bar{e}$, shifts in labour demand result in movements along the relatively flat NSC curve.
     $\Rightarrow$ wages respond less to demand fluctuations, and employment more.

- But the formal model also yields additional insights:
  * The theory implies that **decentralized equilibrium is inefficient**, since the marginal product of labour exceeds the cost of effort.
    $\Rightarrow$ wage subsidies financed by lump-sum taxes improve welfare.

  * The model (modified to allow for flexible hours) also explains **why firms lay off workers** during downturns, rather than reduce hours.
    $\Rightarrow$ Reductions in hours make jobs less valuable, and hence workers would be more inclined to shirk.

- The view that workers are 'rational cheaters' is not uncontroversial – see homework for a more 'humane' theory due to Akerlof and Yellen.

Search and Unemployment

Search and Matching Models

Roadmap:

- Matching in the Labour Market
- The Supply Side: Optimizing Consumer/Workers
- The Demand Side: Optimising Firms
- Equilibrium
- The Beveridge Curve
- Experiments

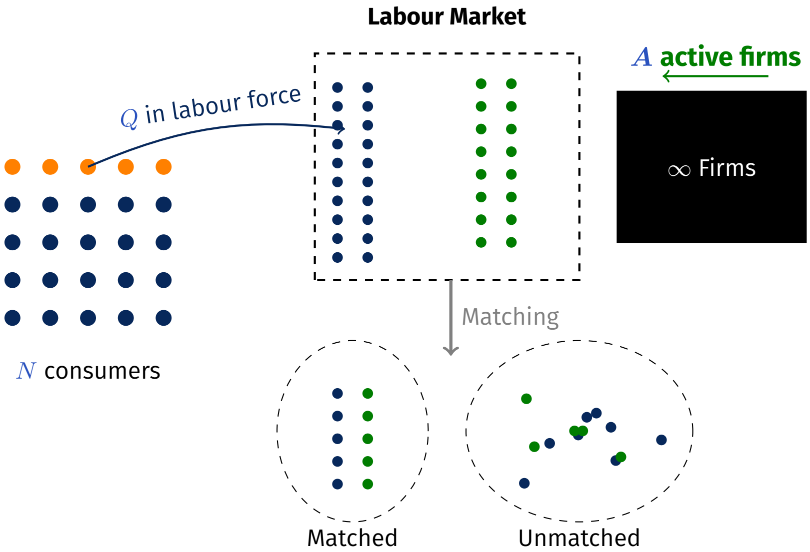
Search and Matching Models

This model is based on the framework introduced by Diamond (1982), Pissarides (1985), and Pissarides and Mortenson (1994)[1].

- The model is *two-sided*, incorporating both the supply side (consumers) and the demand side (firms)
- **Starting point:** workers and jobs are highly heterogeneous.
- Wages are determined by bargaining between workers and firms.
- Matching of workers and jobs occurs through a costly andcomplex process of **search and matching.**

1: Diamond (1982), Pissarides (2000), and Mortensen and Pissarides (1994) are seminal works in labor economics that led to their authors being jointly awarded the Nobel Prize in Economic Sciences in 2010. The prize was given to Peter A. Diamond, Christopher A. Pissarides, and Dale T. Mortensen "for their analysis of markets with search frictions". These economists developed what became known as the Diamond-Mortensen-Pissarides (DMP) model, which has become a fundamental framework for analyzing unemployment and labor markets.

Search and Matching



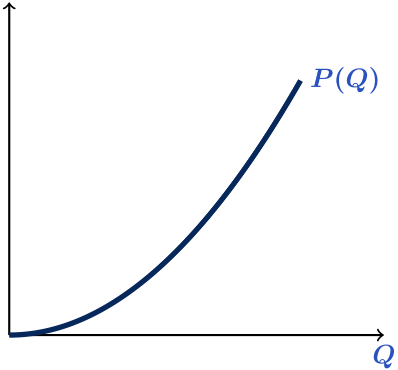
Terminology

- N denotes the working-age population.
- Q represents the labor force (that is, the total number of employed and unemployed individuals).
- A is the number of active firms that are searching for workers.
- The number of firms is determined endogenously within the model.
- U is the number of unemployed persons.
- Unemployment Rate: U/Q
- Participation Rate: Q/N

Workers

- Then, $N - Q$ represents those who remain in home production (i.e., not in the labor force: red area in the figure above).
- We define a supply function, $P(Q)$, which represents the expected payoff from searching for market work.
- The supply curve $P(Q)$ is upward sloping because the opportunity cost of home production varies among consumers.
- A higher expected payoff from searching induces more consumers to forgo home production and participate in the labor market.

Expected Payoff
to Searching for Work



Firms

- In order to produce, a firm must post a vacancy to match with a worker.
- Posting a vacancy is costly; it costs the firm k (measured in units of consumption goods).
- Firms that do not post vacancies remain inactive and are unable to produce.

Matching and the Matching Function

- At the beginning of each period:
  - There are Q consumers actively searching for work.
  - There are A firms posting vacancies - searching for workers.
- Matching workers with firms is a time-consuming and costly process (heterogeneity).
- To capture these difficulties, we use a matching function.
- Let H be the number of successful matches (hiring) between workers and firms.
- Then *the matching function* is defined as

$$H = em(Q, A), \tag{35}$$

  where:
  - e is an efficiency parameter

Matching and the Matching Function

$$H = em(Q, A)$$

- H: Number of matches (output- hires) Q: Workers searching A: Firms with vacancies
- $m(Q, A)$ is like a production function—which produces matches given inputs of workers searching and firms with vacancies.
- e: Matching efficiency (analogous to total factor productivity)
- **Reminder:** Higher e (via better search technologies/information) increases matches.

Properties of the Matching Function

The matching function $\boldsymbol{H = em(Q, A)}$ satisfies the following properties:

1. Non-negativity:
$$\boldsymbol{0 \leq em(Q, A)} \tag{36}$$
2. No matches if either input is zero:
$$\boldsymbol{em(0, A) = em(Q, 0) = 0} \tag{37}$$
3. Matches cannot exceed the minimum of unemployed and vacancies:
$$em(Q, A) \leq \min\left[Q, A\right] \tag{38}$$
4. Increasing but concave in inputs:
$$em_Q(Q, A) > 0, \quad em_A(Q, A) > 0 \tag{39}$$
$$em_{QQ}(Q, A) < 0, \quad em_{AA}(Q, A) < 0 \tag{40}$$
5. **Constant returns to scale**:
$$em(\lambda Q, \lambda A) = \lambda H \tag{41}$$

The Supply Side: Probability of Finding Work

- **Probability of Finding Work (for a Consumer):**

$$p_c = \frac{H}{Q} = \frac{em(Q, A)}{Q} = e\,m\left(1, \frac{A}{Q}\right) \tag{42}$$

where $p_c \in (0, 1)$

  - Equation-42 use the constant returns to scale property of the matching function as in Equation-41.

- **Labor market tightness** is defined as the ratio of firms with vacancies to job seekers.

$$\boldsymbol{j = \frac{A}{Q}} \tag{43}$$

- The probability of finding work is given by:

$$\boldsymbol{p_c = e\,m\,(1, j)} \tag{44}$$

The Supply Side: Probability of Finding Work

$$p_c = e\, m\,(1, j)$$

- By assumption, $m(1, j)$ is increasing in j, so a higher j implies a higher probability of finding work.
- The probability of finding work is increasing in the ratio of vacancies to unemployment.
- Consequently, **the probability of being unemployed** if a consumer chooses to search for work is

$$1 - p_c = 1 - e\, m(1, j) \tag{45}$$

where $(1 - p_c) \in (0, 1)$

  - $1 - p_c$ is decreasing in j
  - Higher labour market tightness implies lower probability of being unemployed. (many firms searching for workers, but few workers searching for jobs)
  - Higher labour market tightness implies higher probability of finding work. (many workers searching for jobs, but few firms searching for workers)

The Supply Side: Expected Payoff to Searching for Work

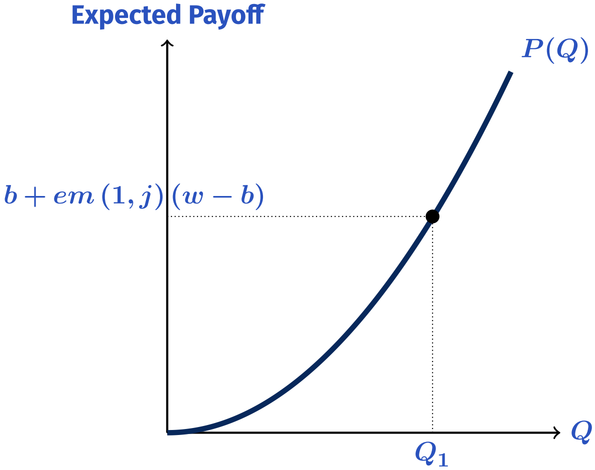
- Recall that $P(Q)$ defines the supply curve for the number of consumers choosing to search for work, Q.
- In equilibrium, $P(Q)$ must be equal to the expected payoff a consumer receives from searching, so
- If consumer work, they receive w, (we will discuss later)
- If they don't work but actively search they still receive b which is the benefit they receive if they are unemployed.
- The expected payoff to searching is

$$P(Q) = p_c w + (1 - p_c)\, b = b + em(1, j)(w - b) \tag{46}$$

- The expression after the second equality is obtained by substituting for p_c using Equation-44.

The Supply Side: Optimization by Consumers

- Figure is an illustration of Equation-46.
- The "market price" for searching workers, or the expected payoff to searching for work on the vertical axis, is determined by the market wage w, the UI benefit b, and market tightness j.
- Then, given this market price, the supply curve for searching workers determines the quantity of searching workers Q.
- A worker in a competitive equilibrium model observes the market wage and then decides how much labour to sell on the market at that wage.
- A worker also takes into account his or her chances of finding work and the unemployment benefit if his or her job search fails.



The Demand Side: Probability of Filling a Vacancy

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Alliance News | 31 July, 2024 | 9:05AM

(Alliance News) - The UK is at risk of having to fight for highly skilled workers in industries such as technology and banking, according to new research.

It is among the top five countries to face a prevalent shortage of talent, recruitment firm Hays PLC said in a report.

Alongside New Zealand, Portugal, Canada and Switzerland, the UK could face major challenges in finding people to fill in-demand and emerging roles in the future.

On the other hand, the US, China, India, Germany and Brazil rank in the top five talent networks across all the sectors it analysed.

Hays said it collected a large global dataset using job adverts and candidate profiles from 31 countries.

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The Demand Side: Probability of Filling a Vacancy

- **Remember:** Firms bear the cost k of posting a vacancy.
- **Probability of filling a vacancy:**

$$p_f = \frac{M}{A} = \frac{em(Q,A)}{A} = e\,m\left(\frac{Q}{A},1\right)$$

where $\boldsymbol{p_f \in (0,1)}$ and $\boldsymbol{j = \frac{A}{Q}}$ is the labor market tightness.

- Then the probability of finding a worker or probability of filling a vacancy:

$$\boldsymbol{p_f = em\left(\frac{1}{j},1\right)} \tag{47}$$

  * The probability of filling a vacancy for a firm is decreasing in labor market tightness $\boldsymbol{j}$.
  * When the labor market is **tight** it is relatively easy for workers to find a job (i.e. p_f is high).
  * Conversely, when the labor market is **slack** it is easy for firms to fill a job, but difficult for workers to find a job (i.e. $\boldsymbol{p_f}$ is high and $\boldsymbol{p_c}$ is low).

The Demand Side: Expected Payoff to Posting a Vacancy

- When a firm is matched with a worker successfully, together they can produce output z where one firm and one worker produce z units of output.
- The firm and worker need to come to an agreement concerning the wage w that the worker is to receive.
- The profit the firm receives from the match is $z - w$, or output minus the wage paid to the worker.
- Firms will enter the labor market, posting vacancies, until the expected net payoff from doing so is zero, or
$$p_f(z - w) - k = 0$$
- Given Equation-47, we can write this equation as
$$em\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

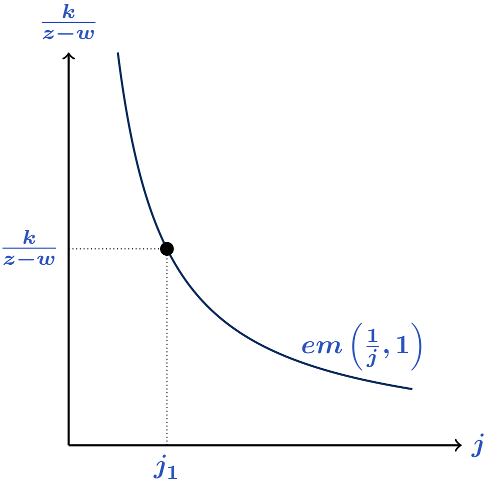
The Demand Side: Optimization by Firms

$$em\left(\frac{1}{j}, 1\right) = \frac{k}{z - w} \qquad (48)$$

- This equation determines labor market tightness j, given the wage w, productivity z, and the cost of posting a vacancy k.
- Firms will enter the labor market and post vacancies, until the expected net payoff from doing so is zero, or

$$p_f(z - w) - k = 0$$

- This is depicted in Figure, where, given $k/(z - w)$, labor market tightness is j_1.



How to Share the Surplus? Let's Bargain!

- Successfull matches generate a surplus:

$$\textbf{Firm Surplus} = z - w \tag{49}$$

$$\textbf{Worker Surplus} = w - b \tag{50}$$

$$\textbf{Total Surplus} = (z - w) + (w - b) = z - b \tag{51}$$

- Need to determine the wage, but this is not a competitive market! In a competitive market, $w = z$.
- In a non-competitive market, the wage is determined by the bargaining power of the worker and the firm.
- **Solution:** Nash bargaining theory in this circumstance dictates that the firm and the worker will each receive a constant share of the total surplus based on their bargaining power.

Solution: Nash Bargaining!

- Let a denote the worker's share of total surplus, where $0 < a < 1$. Here a represents the bargaining power of the worker.

$$w - b = a(z - b) \tag{52}$$

- While the firm's share of total surplus is $(1 - a)(z - b)$.

$$z - w = (1 - a)(z - b) \tag{53}$$

- This is the solution to the Nash bargaining problem:

$$\max\,(w - b)^{a}\,(z - w)^{1-a} \quad \text{s.t.} \quad Total\ Surplus = w - b + (z - w) = z - b \tag{54}$$

- The solution to this problem is given by:

$$\boldsymbol{w = b + a(z - b)} \tag{55}$$

Equilibrium Wage

$$w = b + a(z - b)$$

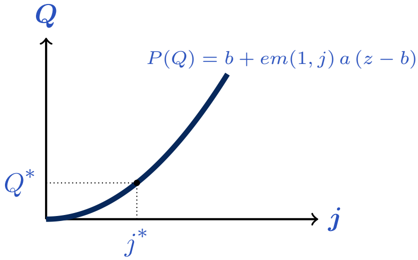
Intuitions:

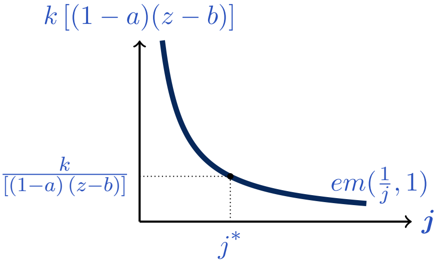
- The equilibrium wage is a linear combination of the output from a match, z, and the household's outside option, b.
- First, we require that $z > b$. If $b > z$, it would never make sense for households to search - the maximum wage they can earn is z, and if their outside option is greater than this, they would never choose to work.
- Second, if $a \to 1$, a firm has no bargaining power, and the equilibrium wage is $w = z$. But if this is the wage, the firm makes a negative profit of k by posting a vacancy.
- In contrast, as $a \to 0$, the firm has all the bargaining power, so workers are paid the minimum required to get them to search, which equals their outside option of b.

Equilibrium in the Labor Market

$$em\left(\frac{1}{j},1\right)=\frac{k}{z-w}=\frac{k}{(1-a)(z-b)} \tag{56}$$

$$P(Q)=b+em(1,j)(w-b) \tag{57}$$





- The first equation determines the equilibrium j given a, k, z and b (exogenous variables).
- Then, given j (endogenous variable), the second equation determines the equilibrium labor force - Q.

Equilibrium Quantities

- **Unemployment Rate:**

$$\boldsymbol{u} = 1 - p_c = \mathbf{1} - \boldsymbol{em}\left(\mathbf{1}, \boldsymbol{j}\right) \tag{58}$$

- **Vacancy Rate:**

$$\boldsymbol{v} = 1 - p_f = \mathbf{1} - \boldsymbol{em}\left(\frac{\mathbf{1}}{\boldsymbol{j}}, \mathbf{1}\right) \tag{59}$$

- **Aggregate Output:**

$$\begin{aligned} Y &= H \times z = z \cdot em\left(Q, A\right) \\ \frac{Y}{Q} &= z \cdot em\left(1, \frac{A}{Q}\right) \\ \boldsymbol{Y} &= \boldsymbol{Q} \cdot \boldsymbol{z} \cdot \boldsymbol{e} \cdot \boldsymbol{m}\left(\mathbf{1}, \boldsymbol{j}\right) \end{aligned} \tag{60}$$

All together: Equations to keep track of

Equilibrium Equations:

$$\begin{cases} em\left(\frac{1}{j},1\right) = \frac{k}{(1-a)(z-b)} \\ P(Q) = b + em(1,j)a(z-b) \end{cases}$$

Outcomes of Interest:

$$\begin{cases} u = 1 - p_c = 1 - em(1,j) \\ v = 1 - p_f = 1 - em\left(\frac{1}{j},1\right) \\ Y = Q \cdot z \cdot e \cdot m(1,j) \end{cases}$$

Connecting Dots: The Beveridge Curve

- For simplicity, we are again going to focus on the **steady state**.
- U is constant, and flows into and out of unemployment balance:
  - $\lambda E = \lambda(1 - U)$ workers become unemployed per unit time, and
  - $p_c U$ workers find employment, allowing us to write:

$$\lambda(1 - U) = p_c U \tag{61}$$

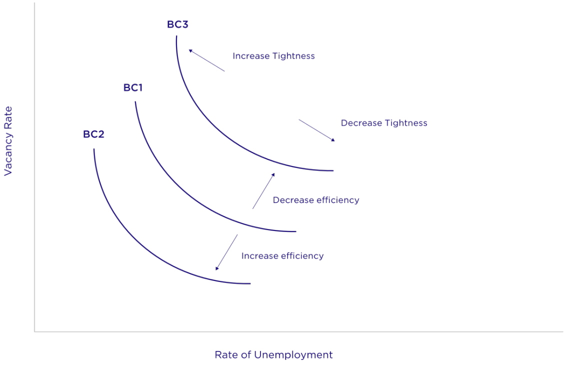
- Using that $p_c = em(1, j)$, where $j \equiv A/Q$, and rearranging yields:

$$U = \frac{\lambda}{\lambda + em(1, A/Q)} = \frac{\lambda}{\lambda + em(1, j)} = \frac{\lambda}{\lambda + p_c} \tag{62}$$

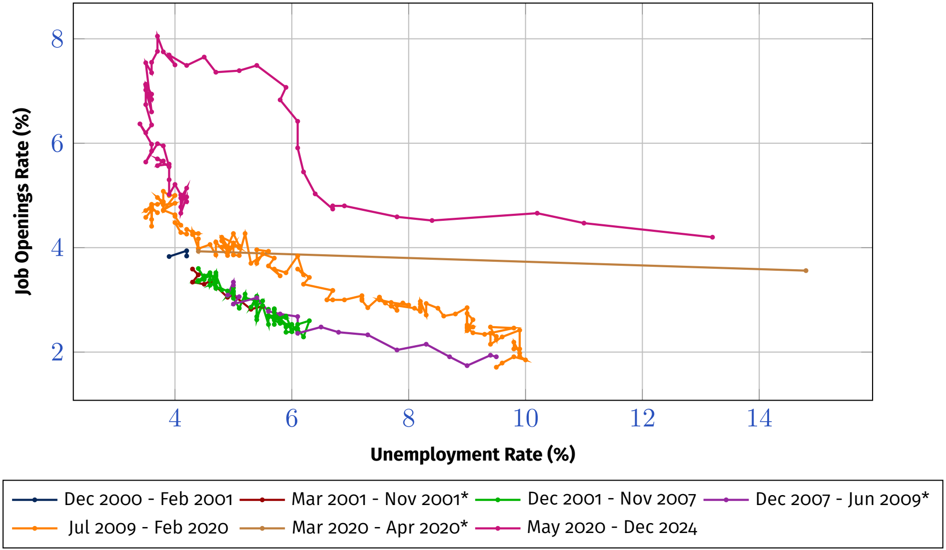
- Equation (62) defines a *negative* relationship between unemployment and vacancies, known the **Beveridge curve**.

The Beveridge Curve

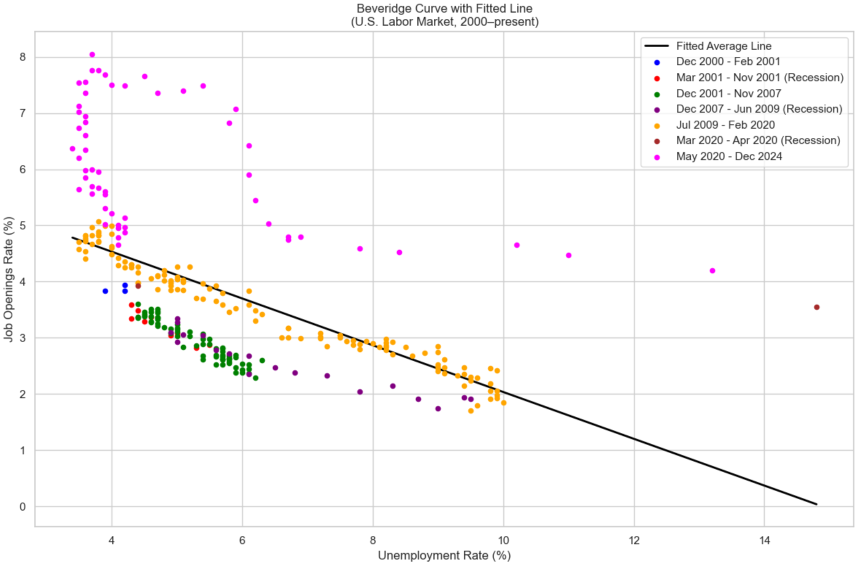
- The Beveridge Curve compares the unemployment rate to the vacancy rate and shows how this changes over time.
- The Beveridge Curve is used to assess the current state of the labour market due to the economic cycle and is also a measure of the efficiency of labour market matching.
- Figure shows how movements along the curve will generally reflect cyclical changes in labour market conditions.
- For example, when the economy strengthens the unemployment rate will fall while job vacancies will rise. When the economy weakens, the opposite is true.
- Firms lay off workers, so unemployment rises and the number of vacancies falls.



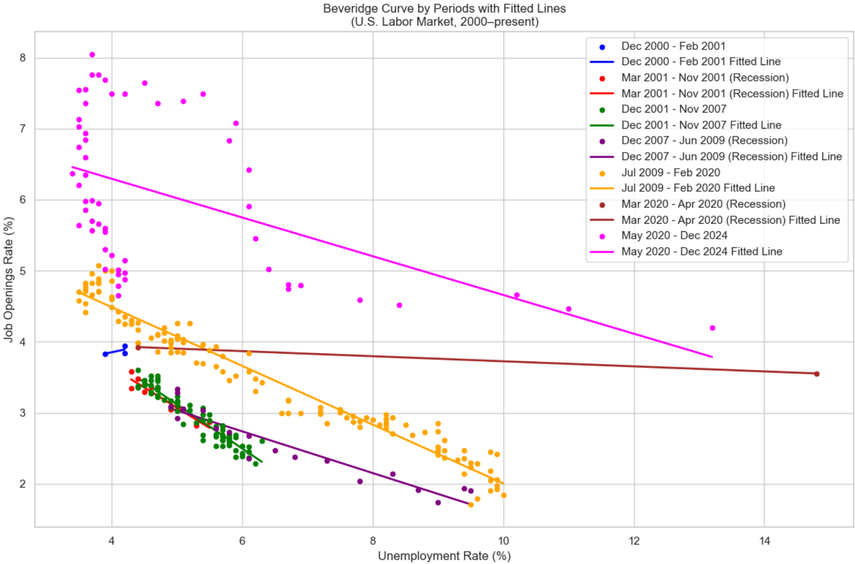
The Beveridge Curve(s) in the US in recent years



The Beveridge Curve

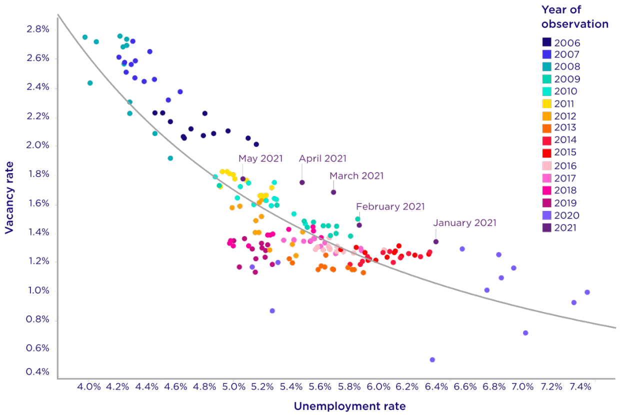


The Beveridge Curve



The Beveridge Curve

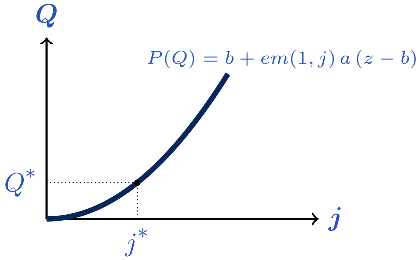
- The Beveridge Curve distinguishes between cyclical and structural changes in the labour market matching process.
- Figure highlights the speed of the shock as well as the subsequent recovery (Covid-19 period).
- The observations over mid-2020 are consistent with a recessionary environment with a relatively high unemployment rate and few vacancies.
- Recent observations (2021) suggest a solid recovery in the labour market with a lower rate of unemployment and an increase in job vacancies.

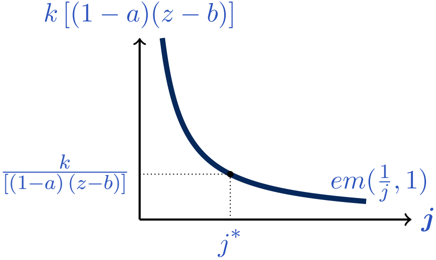


What happens if...

- Productivity, z, increases? **- AI revolution -**
- Efficiency, e, increase/decreases? **- Remote working -**
- Unemployment benefit, b, increases? **- European social state -**
- Cost of posting vacancy, k, decreases? **- Linkedin Effect -**

What would happen if: Experiments





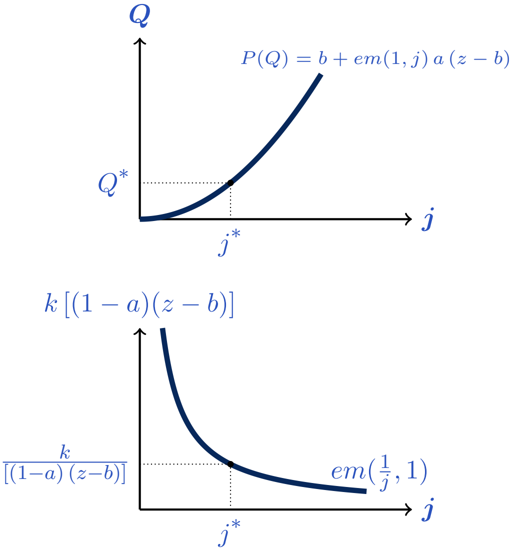
Equilibrium Equations:

$$\begin{cases} em\left(\frac{1}{j}, 1\right) = \frac{k}{(1-a)(z-b)} \\ P(Q) = b + em(1, j)a(z-b) \end{cases}$$

Outcomes of Interest:

$$\begin{cases} u = 1 - p_c = 1 - em(1, j) \\ v = 1 - p_f = 1 - em\left(\frac{1}{j}, 1\right) \\ Y = Q \cdot z \cdot e \cdot m(1, j) \end{cases}$$

What would happen if: Experiments



- The smaller is the cost of posting a vacancy relative to the firm's share of total surplus, $k/(1 - a)(z - b)$, the higher job market tightness, j will be.
- If labor market tightness j is higher, then the chances of finding a job are greater for consumers, more of them will decide to search for work, and therefore Q will be higher.
- For example, in Figure higher j increases the expected payoff to searching for work, and then a higher supply of searching workers, Q , is forthcoming.