Budget Deficits and Fiscal Policy

- WEEK EIGHT -

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https://macroeconomics.info

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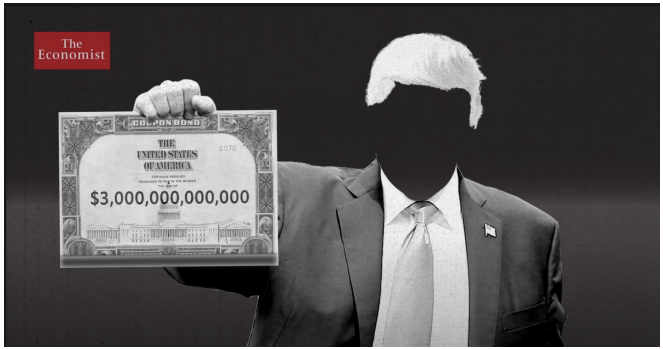
Questions

Is the US in trouble in terms of debt?

Source: Economist https://www.doge.gov/savings

- In 2023, the U.S. government paid a record $658B in debt service, marking the first time in history that spending on debt interest payments exceeded defense spending.
- 2024, the U.S. federal government spent $6.75T while collecting $4.92T in revenue, resulting in a deficit of $1.83T.
- As of early 2025, the U.S. public debt has surpassed $36T, exceeding 120% of the country's (GDP)
- As of 2025, the U.S. share of global debt is 35%.
- and new government created the Department of Government Efficiency (DOGE).

How much is too much debt?



https://www.eiu.com/n/campaigns/sovereign-debt-crises-are-coming/

https://foreignpolicy.com/2020/10/20/election-2020-global-debt-crisis/

The importance of sovereign debt

- Sovereign states have borrowed money for hundreds of years.
- Sovereign debt is one of the largest classes of financial assets: 19% of the world total financial assets in 2010.

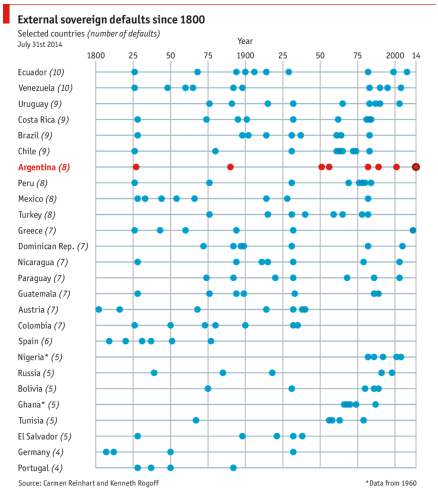
Sovereign Debt as a Share of All Financial Assets

	Listed in London				London and Some Foreign			World			
	1853[a]	1873[a]	1893[a]	1913[a]	1933[a]	1950[a]	1950[b]	1950[c]	1978[c]	1990[d]	2010[d]
Total Assets	£1.2b	£2.3b	£4.9b	£11.2b	£18.5b	£15.2b	£14.3b	--	--	$54t	$212t
Public Debt	76%	59%	39%	35%	59%	78%	60%	22%	11%	17%	19%
Of which: UK	70%	38%	18%	14%	38%	66%	57%	--	--	--	--
Foreign & Colonial	6%	21%	21%	21%	21%	11%	3%	--	--	--	--

[a]Data from *Stock Exchange Official Intelligence* as reported in Michie (2001) Tables 3.2 and 5.1. All securities at face value. Data refer to securities listed on the London Stock Exchange, except for 1933 and 1950 which includes foreign and colonial public sector securities listed abroad.
[b]Data from *Stock Exchange Official Intelligence* as reported in Michie (2001) Table 8.1. All securities at market value. Data refer to securities listed on the London Stock Exchange, plus foreign and colonial public sector securities listed abroad.
[c]Data from Goldsmith (1985) Table 1 on share of government debt in financial assets, all measured at market values.
[d]Data from Roxburgh, Lund and Piotrowski (2011) Exhibit E1 on public debt securities at face value, relative to other debt at face value and equities at market value.

Source: Tomz and Wright (2013)

Is the sovereign debt crisis a rare event?



- Between 1820 and 2012, 251 defaults by 107 entities.
- Most frequent defaulters: Ecuador, Mexico, Uruguay, Venezuela (+ 8 episodes each); serial defaulters (Reinhart and Rogoff, 2004).
- Largest defaults: Greece 2012 - $ 261bn, Argentina 2002 - USD132bn.
- Unconditional probability of default is 1.8% per year:
  - Extremely sensitive to definition of default, particularly how different episodes are aggregated.
  - Arteta and Hale (2008): probability of default of 13 % per year.

Government Debt Dynamics and Ricardian Equivalence

Introduction to Fiscal Policy

- **Fiscal policy** involves government decisions regarding levels and composition of **taxation** and **public spending**.
- **Direct effect**: In contrast to monetary policy (interest rates, money supply), fiscal policy directly impacts aggregate demand through spending (G) and taxes (T).
- **Political process**: Fiscal decisions are influenced by political processes, making rapid policy adjustments challenging compared to monetary policy.
- **Consensus**: Stabilization policies are better managed by monetary authorities.
  - Quick implementation
  - Less political interference
- **Importance of Fiscal Policy**: Even if monetary policy is optimal for short-term stabilization, fiscal policy profoundly influences long-run economic growth, income distribution, and efficiency. For instance, distortionary taxes may discourage work or investment, while well-structured government spending (e.g., infrastructure, education) can boost economic growth.

Government Debt Dynamics

Two periods

- **Period 1**: The government finances spending through either taxes or debt

$$G_1 = T_1 + D$$

- **Period 2**: The government finances spending through taxes

$$G_2 + (1 + r)D = T_2$$

Multiple Peirods (N)

- Generilise this into a multi-period discrete model for period $t = 0, 1, 2, \ldots, N$

$$G_t + (1 + r)D_{t-1} = T_t + D_t$$

Continuous time

$$G_t + (r)D_t = T_t + \dot{D}_t \quad \longrightarrow \quad \dot{D}_t = G_t + rD_t - T_t \tag{1}$$

- $\dot{D}_t$ is the change in debt at time t.
- Debt accumulates whenever government spending (+ interest payments) exceeds taxes.

Government Debt Dynamics (+)

- Note that $D(t)$ denotes the **level (or stock)** of government debt outstanding at time t.
- $D(t)$ represents the **stock of accumulated debt** that the government owes at precisely moment t.
- $\dot{D}(t)$, which is the derivative of $D(t)$ with respect to time, represents the **change (or flow)** of debt at time t.
  - If $\dot{D}(t) > 0$, debt increases (the government is borrowing more).
  - If $\dot{D}(t) < 0$, debt decreases (the government is repaying debt).
- Thus, the term $rD(t)$ specifically means the **interest payments** on the existing level (stock) of debt at time t.

Government Budget Constraint (Lifetime)

- Integrating equation-1 from the present moment 0 into the infinite future, discounting each term by e^{-rt} and after considering transversality condition, and rearranging to isolate taxes explicitly, we get the final government lifetime budget constraint:

$$\int_0^\infty e^{-rt} T(t)dt = D(0) + \int_0^\infty e^{-rt} G(t)dt \tag{2}$$

For detailed explanation go to https://macroeconomics.info/.

- The present discounted value of future tax revenues ($T(t)$) must exactly cover two components: **1. Initial debt $D(0)$** and **2. The present discounted value of future government expenditures $G(t)$**.
  - Government borrowing today does not permanently avoid taxes; it merely shifts taxes from the present to the future.
  - Future taxpayers must eventually finance past and present spending decisions through higher taxes, aligning closely with **Ricardian Equivalence** intuition.

Government: It is all about tax

- **How does the government finance itself?**
  1. Current tax income, or
  2. Borrowing against future tax incomes
  - If the government chooses to borrow, it will need to repay the debt in the future, which will require higher taxes.
  - If the government chooses to tax today, it will reduce the amount of resources available for private consumption and investment.
- **How does this choice affect the household consumption?**
- **Ricardian Equivalence:** It does not matter! (Robert Barro (1974))
- Rational and forward-looking households increase savings to exactly offset the expected future tax burden, leaving consumption unaffected. (if govt chooses to borrow)
- **How?**

Ricardian Equivalence: 2 Periods

Household Budget Constraint:

Period 1: Household income is Y_1, consumption is C_1, taxes are T_1, and saving is S:

$$Y_1 - T_1 = C_1 + S$$

Period 2: Household receives income Y_2, pays taxes T_2, consumes C_2, and earns interest on savings:

$$Y_2 - T_2 + (1 + r)S = C_2$$

Combining both periods we can get:

$$C_1 + \frac{C_2}{1+r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1+r} \tag{3}$$

Government Budget Constraint:

Period 1: Government finances spending G_1 via taxes T_1 or debt issuance D:

$$G_1 = T_1 + D$$

Period 2: Debt and interest must be paid from taxes collected:

$$G_2 + (1 + r)D = T_2$$

Ricardian Equivalence: 2 Periods

Ricardian Equivalence in Household Decision-Making:

- Using the government constraints, households see clearly that a decrease in taxes today ($T_1 \downarrow$) implies an increase in taxes tomorrow ($T_2 \uparrow$).
- Thus, household lifetime resources remain unchanged:

$$(Y_1 - T_1) + \frac{Y_2 - [G_2 + (1+r)(G_1 - T_1)]}{1+r} = Y_1 + \frac{Y_2 - G_2 - (1+r)G_1}{1+r}$$

Conditions for Ricardian Equivalence:

- Lump-sum, non-distortionary taxes
- Perfect capital markets (households borrow/save at government interest rates)
- Rational, forward-looking households

Ricardian Equivalence: Infinite Periods

Consider a household with earning income $Y(t)$, paying taxes $T(t)$, consuming $C(t)$, and accumulating savings with an interest rate r and no initial wealth. The household's intertemporal (lifetime) budget constraint in continuous time is:

$$\int_0^\infty e^{-rt} C(t) dt \leq \int_0^\infty e^{-rt} [Y(t) - T(t)] dt$$

We have taxes explicitly from the government's budget constraint as:

$$\int_0^\infty e^{-rt} T(t)\, dt = D(0) + \int_0^\infty e^{-rt} G(t)\, dt$$

Substituting this explicitly into the household constraint, we clearly obtain:

$$\int_0^\infty e^{-rt} C(t)\, dt \leq \int_0^\infty e^{-rt} Y(t)\, dt - \left[D(0) + \int_0^\infty e^{-rt} G(t)\, dt \right]$$

Let's rewrite explicitly and clearly simplify the terms:

$$\boxed{\int_0^\infty e^{-rt} C(t)\, dt \leq \int_0^\infty e^{-rt} Y(t)\, dt - D(0) - \int_0^\infty e^{-rt} G(t)\, dt} \tag{4}$$

This equation clearly indicates that household lifetime consumption depends only on the present discounted value of their income and government expenditures, not on how the government finances expenditures (tax vs. debt).

Ricardian Equivalence: Limitations

Ricardian equivalence rarely holds perfectly in reality. Three main reasons for its failure include:

- **Distortionary taxation:**
  - Real-world taxes distort economic choices (e.g., labor supply, investment), and thus tax timing matters significantly for behavior.
- **Liquidity constraints:**
  - Many households can't borrow freely and/or at the same low interest rate as the government.
  - Debt-financed tax cuts increase their disposable income directly and thus consumption rises, since they cannot borrow easily to smooth consumption on their own.
- **Precautionary saving and uncertainty:**
  - When future taxes and incomes are uncertain, households do not adjust consumption fully rationally, which breaks the Ricardian neutrality.
- **Empirical Example (Real-World Application):**
  - The 2008–2009 U.S. tax cuts during the global financial crisis aimed explicitly at stimulating current consumption spending.
  - Empirical evidence shows that many households increased consumption in response, contradicting pure Ricardian equivalence.

Tax Smoothing: Optimal Taxation

Cost of Distortionary Taxes: Reasons

Most taxes are **distortionary**—they alter people's behavior and economic decisions. Specifically:

- **Labour Income Taxes**:
  - Discourage work effort or reduce the incentive to supply labor (workers may choose leisure over additional taxed income).
- **Capital Income Taxes**:
  - Discourage saving and investment by reducing the after-tax returns on capital.
- **Value-Added Taxes (VAT) and Consumption Taxes**:
  - Distort consumption decisions by changing relative prices of goods and services, affecting consumer choices.
- These distortions introduce economic inefficiencies, reducing overall welfare and potentially hindering economic growth.

Cost of Distortionary Taxes: Optimal Taxation

- Government finances predetermined spending G_1, G_2.
- Finances via taxes T_t or borrowing at interest rate r.
- Government budget constraint:

$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

- The optimal tax rate satisfies:

$$\text{Marginal social cost of taxation} = \text{Marginal social benefit of taxation}$$

- **Resource Constraint** (Economy-wide): (zero net export and no investment)

$$Y_t = C_t^{private} + G_t + \zeta_t$$

- Where ζ_t is the distortionary tax cost:

$$\zeta_t = Y_t f\left(\frac{T_t}{Y_t}\right)$$

Cost of Distortionary Taxes: Optimal Taxation

$$\zeta_t = Y_t f\left(\frac{T_t}{Y_t}\right), \quad f(0) = 0, \quad f'(.) > 0, \quad f''(.) > 0$$

- ζ_t represents the **total cost** (in terms of lost output or welfare) due to **distortionary taxes** at time t.
- The ratio $\frac{T_t}{Y_t}$ is the **tax rate** expressed as a proportion of the economy's total output or income. This normalized tax rate is critical because distortionary costs typically depend on **how high** taxes are relative to the economy's overall size.
- $f\left(\frac{T_t}{Y_t}\right)$ represents the distortionary cost **per unit of output** as a function of the tax rate.
- Thus, the distortionary cost per unit of output is scaled by total output Y_t to get **total distortionary costs** in absolute terms:

$$\text{Distortionary cost per unit output} \times \text{Total output} = \text{Total distortionary cost}$$

  - Higher $\frac{T_t}{Y_t}$ (taxes represent a larger share of output) $\rightarrow$ stronger economic distortions.
  - Lower $\frac{T_t}{Y_t} \rightarrow$ milder distortions.

Cost of Distortionary Taxes: Optimal Taxation

- The government's problem can be expressed as:

$$\min_{T_1,T_2} \left[Y_1 f\left(\frac{T_1}{Y_1}\right) + \frac{Y_2}{1+r} f\left(\frac{T_2}{Y_2}\right) \right]$$

- subject to its budget constraint:

$$\boldsymbol{T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}}$$

- The government seeks to minimize the total discounted distortionary cost across both periods subject to the budget constraint above.

Cost of Distortionary Taxes: Optimal Taxation

- Define the Lagrangian function and take FOCs w.r.t T_1 and set to zero:

$$\frac{\partial \mathcal{L}}{\partial T_1} = Y_1 f'\left(\frac{T_1}{Y_1}\right)\frac{1}{Y_1} - \lambda = 0$$

- Simplifies explicitly to:

$$f'\left(\frac{T_1}{Y_1}\right) = \lambda \tag{5}$$

- With respect to T_2:

$$\frac{\partial \mathcal{L}}{\partial T_2} = \frac{Y_2}{1+r} f'\left(\frac{T_2}{Y_2}\right)\frac{1}{Y_2} - \frac{\lambda}{1+r} = 0$$

- Again simplifying explicitly:

$$f'\left(\frac{T_2}{Y_2}\right) = \lambda \tag{6}$$

Cost of Distortionary Taxes: Optimal Taxation

- From the FOCs (equations 5 and 6), we directly see that:

$$f'\left(\frac{T_1}{Y_1}\right) = f'\left(\frac{T_2}{Y_2}\right) = \lambda \tag{7}$$

For detailed explanation go to https://macroeconomics.info/.

- **Marginal distortionary cost** (deadweight loss per additional unit of revenue raised) must be equalised across both periods.
- At optimum, no further gains in efficiency are possible through intertemporal reallocation.
- This explicitly ensures the **efficiency** of intertemporal taxation.
- Since f is strictly convex, equation 7 can only hold if the **tax rate** (i.e. tax as a share of output) is constant across periods:

$$\frac{T_1}{Y_1} = \frac{T_2}{Y_2} \tag{8}$$

Optimal Taxation: Implications

$$\frac{T_1}{Y_1} = \frac{T_2}{Y_2}$$

- The core result of tax smoothing theory highlights that a rational, welfare-maximising government aims to keep tax rates stable across periods.
- Frequent and significant variations in taxes are economically costly and inefficient.
- Optimal fiscal policy usually involves stable, predictable taxation rates rather than volatile short-term adjustments.
- Policymakers thus strive for consistent, stable taxation frameworks, which help promote economic efficiency, long-term growth, and fiscal sustainability.

But how does the government manage temporary and unexpected expenditures? Like wars and recessions.

Optimal Taxation: Implications

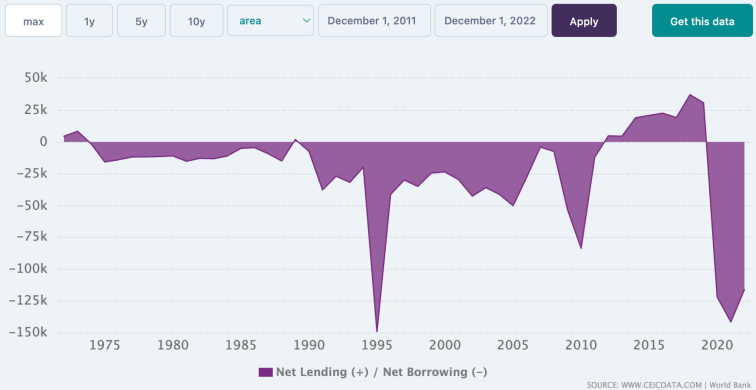
- If governments faced temporary expenditure shocks and tried to finance these entirely through current taxes, this would cause substantial tax increases, sharply raising marginal distortion costs.
- To avoid such costly disruptions, the optimal strategy for governments is borrowing.

Why Borrowing?

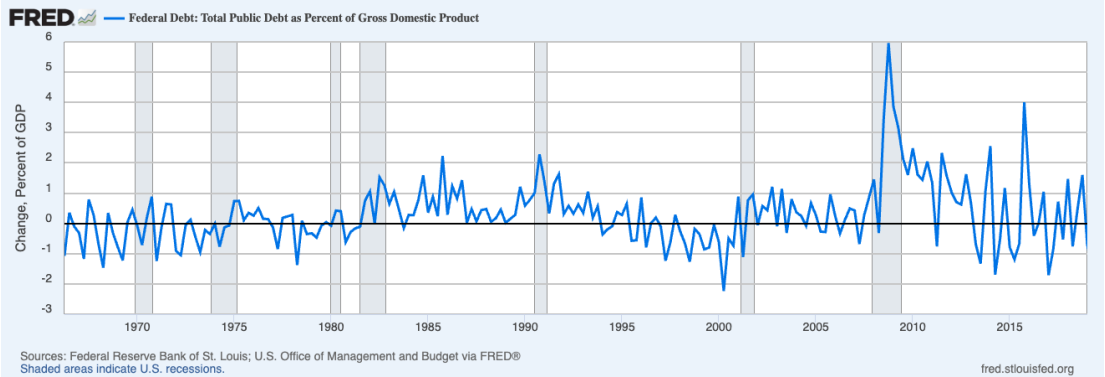
- Borrowing allows governments to smooth out temporary spikes in expenditures over several periods, keeping tax rates stable and avoiding dramatic short-run tax increases.
- By spreading out repayment over the future (when expenditure returns to normal), borrowing significantly reduces total economic inefficiencies.
- **How does this work in practice?**

Debt and Deficits

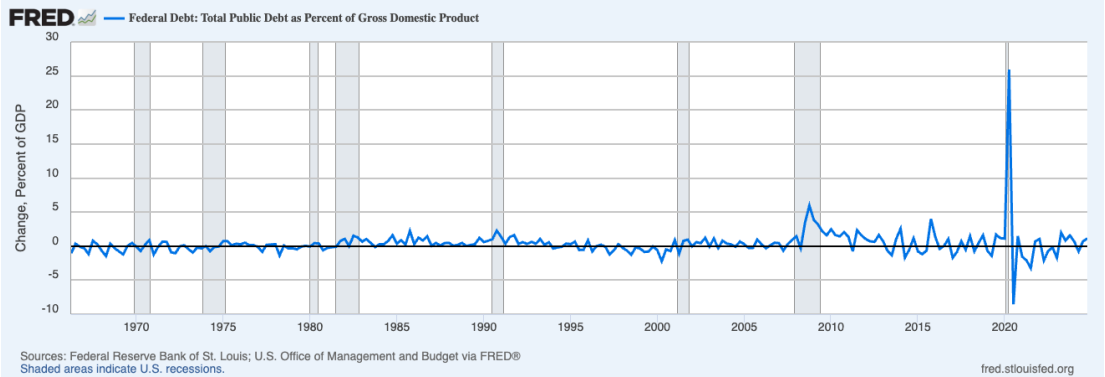
View Germany's Germany DE: Net Lending (+) / Net Borrowing (-) from 1972 to 2022 in the chart:



Debt and Deficits



Debt and Deficits



Debt Stabilisation

Not temporary but persistent

- While tax smoothing can justify short-run deficits (e.g., wars, recessions), it does not naturally explain **why some countries continually run deficits even in good economic times**.
- Persistent deficits imply that the government is continually borrowing, adding to debt levels every year, rather than just borrowing temporarily.
- Persistent deficits can reflect structural problems or political incentives, rather than optimal fiscal strategies.
- For example the U.S. have run large deficits consistently over several decades.
  - **Auerbach & Gale (2017)** estimated that if the U.S. maintains its current fiscal policy unchanged, the government would need to raise taxes by approximately **7.7% of GDP** (relative to the existing tax level of around **18% of GDP**) to satisfy its long-term budget constraint.
  - This substantial gap signals severe fiscal imbalance and potential unsustainability, reflecting structural mismatches between spending commitments (e.g., pensions, healthcare, defense) and tax revenues.

Public Debt, Deficit, and Beyond

- All variables are expressed in real terms (goods).
- Define clearly:
  - Government spending in period t: G_t
  - Real interest rate: r_t
  - Taxes in period t: T_t
- Primary deficit in period t: $G_t - T_t$
  - If $G_t - T_t > 0$, government runs a deficit and must borrow and let say this is B_t.
- Law of motion of government debt:

$$B_t = (1 + r_t) B_{t-1} + G_t - T_t$$

- Then, we can write the budget deficit ($\mathcal{D}$) in year t as

$$Deficit(\mathcal{D}_t) = \Delta B_t = B_t - B_{t-1} = \underbrace{r_t B_{t-1}}_{\text{Interest Payments}} + \underbrace{G_t - T_t}_{\text{Primary Deficit}}$$

Public Debt, Deficit and Beyond

$$\mathcal{D}_t = \Delta B_t = B_t - B_{t-1} = r_t B_{t-1} + G_t - T_t$$

- The total deficit, which is equal to the change in government debt ΔB_t, is equal to the sum of interest payments and the primary deficit $G_t - T_t$
- B_{t-1} is government debt at the end of year $t-1$, or, equivalently, at the beginning of year t;
- r is the real interest rate, which we shall assume to be constant here.
- Thus, rB_{t-1} equals the real interest payments on the government debt in year t.

Do not confuse the words **deficit** and **debt**. Debt is a stock— what the government owes as a result of past deficits. The deficit is a flow—how much the government borrows during a given year.

A Fiscal Policy: Decreasing Tax in Year 1

- Initial condition (year 0): Balanced budget $(G = T)$, zero debt $(\mathcal{B}_0 = 0)$.
- Policy intervention:
  - Year 1: Taxes reduced temporarily by 1 unit; spending remains constant.
  - Immediate consequence: debt rises to $\mathcal{B}_1 = 1$.

Central Question:

- **How will this temporary tax cut impact future fiscal conditions?**

Repayment in Year 2

- Government's budget constraint (year 2):

$$\mathcal{B}_2 = (1+r)\mathcal{B}_1 + (G_2 - T_2) \tag{9}$$

- Condition for full debt repayment at year-end ($\mathcal{B}_2 = 0$):
- Replacing $\mathcal{B}_1$ by 1 and $\mathcal{B}_2$ by 0 and transposing terms gives

$$T_2 - G_2 = (1+r)\mathcal{B}_1 = (1+r)$$

Intuition:

* To eliminate debt incurred from a temporary tax cut, the government must generate a primary surplus exactly equal to debt plus interest.

- **How to achieve this surplus?**

A Fiscal Policy: Decreasing Tax in Year 1

- Two possible adjustments:
  1. Reduce government spending (G).
  2. Increase taxes (T).
- Assuming government spending remains unchanged:
  - Required tax increase in year 2 is $(1 + r)$.
- Thus, a 1-unit tax cut in year 1 necessitates a larger future tax increase of $(1 + r)$.

Key Insight:

- Temporary fiscal relief today implies larger fiscal burdens tomorrow.

Repayment in Year t

- If repayment is delayed until year t, and from year 2 to $t-1$, primary deficit = 0:

$$B_{t-1} = (1+r)^{t-2}$$

- **How?**
- From year 2 to year $t-1$ the primary deficit is equal to zero; so, debt at the end of year 2 is:

$$\mathcal{B}_2 = (1+r)\mathcal{B}_1 + 0 = (1+r)1 = (1+r)$$

- With the primary deficit still equal to zero during year 3 , debt at the end of year 3 is

$$\mathcal{B}_3 = (1+r)\mathcal{B}_2 + 0 = (1+r)(1+r)1 = (1+r)^2$$

- **Debt growth without repayment:**
  - Even without new primary deficits, existing debt grows exponentially due to accumulating interest.

Repayment in Year t

- Year t budget constraint (debt fully repaid at end-year):

$$0 = (1 + r)B_{t-1} + (G_t - T_t)$$

- Substitute $B_{t-1} = (1 + r)^{t-2}$:

$$T_t - G_t = (1 + r)^{t-1}$$

Key Implications:

* The longer the delay or the higher the interest rate, the greater the eventual required surplus.
* Temporary relief becomes increasingly expensive over time.

Dont pay but stabilize it

- Debt stabilization: Keeping debt constant at existing level from a certain period onwards.
- For stabilization starting at year 2 ($\mathcal{B}_2 = \mathcal{B}_1 = 1$):

$$1 = (1 + r)\mathcal{B}_1 + (G_2 - T_2)$$

- Primary surplus required:

$$T_2 - G_2 = r$$

Intuition (?):

- To stabilize debt, the government must run a primary surplus exactly equal to interest payments every year.

Dont pay but stabilize it

- Stabilizing rather than repaying debt implies:
  - Permanently higher debt due to past deficits.
  - Permanent increase in taxes (or decrease in spending) equal to annual interest payments.

Conclusions:

- Past deficits have lasting effects; stabilizing debt requires permanent fiscal adjustments.
- Temporary deficits can lead to permanently higher taxation or reduced public services.

Summary – Key Lessons on Debt Dynamics

- Debt dynamics depend critically on:
  - Real interest rate relative to economic growth.
  - Fiscal policy (primary deficit or surplus).

  Key Takeaways:
  - Short-term deficits create long-term fiscal burdens.
  - Delaying fiscal adjustments amplifies future costs.
  - Debt stabilization requires continuous fiscal discipline.
- Understanding these dynamics is essential for sustainable fiscal policymaking.

Unsustainable Debt

Law of motion for (or, The Evolution of) Public Debt

- Fundamental debt equation:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

- Debt-to-GDP ratio evolution:

$$\frac{B_t}{Y_t} = \frac{(1 + r_t)B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

- Define debt-to-GDP ratio clearly as:

$$b_t \equiv \frac{B_t}{Y_t}$$

- Thus:

$$b_t = \frac{(1 + r_t)Y_{t-1}}{Y_t} b_{t-1} + \frac{G_t - T_t}{Y_t}$$

- Assume GDP growth rate g_Y, hence:

$$\frac{Y_t}{Y_{t-1}} = 1 + g_Y$$

Law of motion for (or, The Evolution of) Public Debt

- Debt-to-GDP ratio equation becomes:

$$b_t = \frac{1 + r_t}{1 + g_Y} b_{t-1} + \frac{G_t - T_t}{Y_t}$$

Insights on dynamics:

- If primary deficit $(G_t - T_t) = 0$, absolute debt grows at rate r.
- However, debt-to-GDP grows at rate $r - g_Y$.

Important determinants of debt-to-GDP ratio growth:

- Higher real interest rate (r_t) accelerates debt accumulation.
- Lower GDP growth rate (g_Y) makes debt-to-GDP ratio increase faster.
- Higher initial debt ratio increases sensitivity to interest-growth differential.
- Larger primary deficits further drive up the ratio.

Maths

Useful approximation:

$$\frac{1+r}{1+g} \approx 1 + r - g$$

Proof (Intuition):

- Consider:

$$(1 + r - g)(1 + g) = 1 + r + rg - g^2$$

- If r and g are small, products rg and g^2 become negligible:

$$(1 + r - g)(1 + g) \approx 1 + r$$

- Dividing by $1 + g$ yields:

$$\frac{1+r}{1+g} \approx 1 + r - g$$

Numerical Example:

* If $r = 0.05, g = 0.03$, exact: 1.019; approximate: 1.02 (close approximation).

Sustainability of Public Debt

Long-term sustainability analysis:

- Assume zero primary deficits beyond time t_0: $G_t = T_t$ for all $t \geq t_0$.
- Constant real interest rate $r_t = r$ for $t \geq t_0$.
- Debt-to-GDP ratio evolves as:

$$b_t = \frac{1+r}{1+g_Y} b_{t-1}$$

- Thus, explicitly:

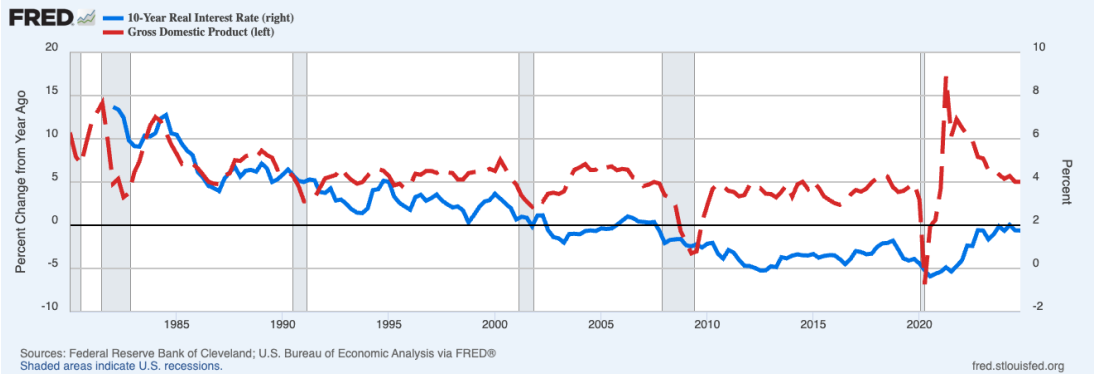
$$b_t = (1 + r - g_Y)^{t-t_0} b_{t_0}$$

Three critical cases:

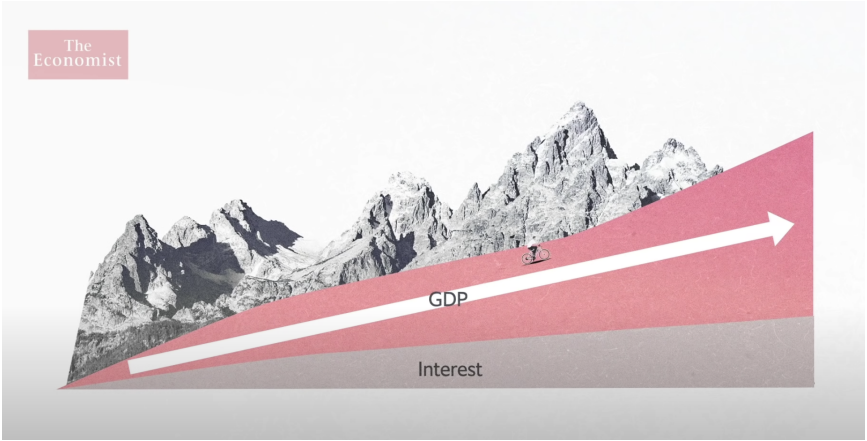
- If $r < g_Y$: Debt-to-GDP converges to zero (dynamic inefficiency scenario).
- If $r = g_Y$: Debt-to-GDP remains constant (borderline sustainability).
- If $r > g_Y$: Debt-to-GDP ratio grows exponentially without bound, indicating unsustainability.

Crucial Insight: The sustainability of debt depends fundamentally on the interest-growth differential $(r - g_Y)$.

Some Questions: if $g > r$

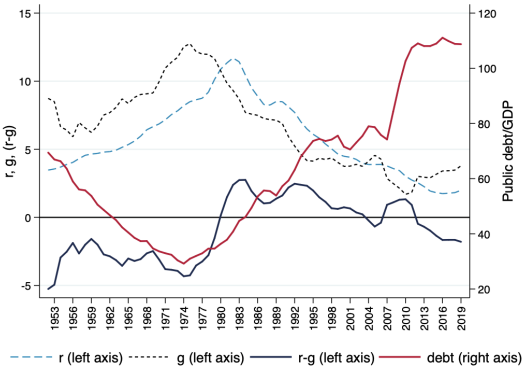


Some Questions: if $g > r$



Is public debt sustainable in the world?

Source: IMF Working Paper WP/20/137: *Public Debt and r - g at Risk* by Weicheng Lian, Andrea F. Presbitero, and Ursula Wiriadinata



This figure plots the time series of the world's interest rate r, growth rate g, interest-growth differential $r - g$, and public debt-to-GDP ratio based on 17 advanced economies over the period 1950-2019. The world's values are GDP-weighted averages. All variables are 5 -year moving averages. $r - g_t$ are computed as nominal local currency long-term rates r_t minus nominal local currency annual growth rates g_t.

Cost of Persistent Deficits

Persistent deficits have several significant economic costs:

- **Higher interest payments:**
  - Increased debt requires greater interest payments, reducing resources for productive public investments.
- **Reduced national saving and investment (crowding out):**
  - Government borrowing reduces funds available for private investments, potentially harming long-term growth.
- **Fiscal unsustainability and default risk:**
  - Unsustainable deficits lead to higher borrowing costs, risk sovereign debt crises, and erode investor confidence.
- **Increased future tax burden:**
  - Current deficits inevitably demand future tax increases or spending cuts, burdening future generations and distorting economic incentives.

Cost of unsustainable debt

Unsustainable debt occurs when $r > g_Y$:

$$b_t = (1 + r - g_Y)^{t-t_0} b_{t_0}$$

Consequences of unsustainability:

- **Sovereign debt crisis:** Sudden loss of investor confidence and sharp rise in borrowing costs.
- **Severe fiscal adjustments:** Required austerity measures (spending cuts, higher taxes) leading to economic contractions.
- **Economic disruptions:**
  - Financial crises (banking instability)
  - Exchange rate crises (currency depreciation)
  - Twin crises (simultaneous financial and currency crises)

Strategic Debt Accumulation

Deficit Bias – Political Economy Overview

- Governments often run persistent deficits despite clear economic costs, a phenomenon known as **deficit bias**.
- Such deficits typically arise due to political rather than purely economic considerations.
- We examine two key political-economic explanations:
  - **Strategic debt accumulation** (Tabellini and Alesina, 1990)
  - **Delayed fiscal stabilization** due to bargaining conflicts (Alesina and Drazen, 1991)

Strategic Debt Accumulation – Setup

- Two-period economy (periods 1 and 2).
- Government receives fixed endowment W each period.
- Two types of public spending: goods M and N.
  - Example: Military vs. Healthcare
- Period-1 policymaker:
  - Chooses spending on M_1, N_1, and amount of debt D.
  - Budget constraint:

$$M_1 + N_1 = W + D$$

- Period-2 policymaker faces budget constraint:

$$M_2 + N_2 = W - D$$

- Borrowing $(D > 0)$ increases current spending at future expense.

Polarized Preferences – The Strategic Element

- Two types of policymakers with opposing preferences:
  - **Type-1**: Only values good M (utility $U(M)$).
  - **Type-2**: only values good N: utility $U(N)$.
  - Standard assumptions on utility: $U' > 0, U'' < 0$.
- Type-1 policymaker in period 1 maximizes spending on good M:

$$M_1 = W + D, \quad N_1 = 0$$

- Period-2 policymaker uncertain:
  - Probability π: Type-1 continues spending on M.
  - Probability $1 - \pi$, Type-2 shifts spending entirely to good N.

Optimal Debt Choice and Strategic Behavior

- Type-1 policymaker maximizes expected utility by choosing debt D:

$$\max_{D} \; U(W+D) + \pi U(W-D) + (1-\pi)U(0)$$

- First-order condition for optimality:

$$\frac{U'(W+D)}{U'(W-D)} = \pi$$

- Analysis:
  - If future policymaker is certainly Type-1 ($\pi = 1$), optimal debt $D = 0$.
  - If future policymaker uncertain $(\pi < 1)$, optimal debt is $D > 0$, strategically limiting future policymaker's choices.
  - The lower π, the higher the incentive to accumulate debt.

Intuition behind Strategic Debt Accumulation

- Strategic debt arises because current policymakers prefer to allocate resources according to their priorities, anticipating potential misallocation by future policymakers.
- Debt is strategically used as a commitment mechanism to transfer resources from an uncertain future to the current period, aligning resource use with current policymaker preferences.

Economic Consequences:

- Inefficient intertemporal allocation of resources.
- Persistent deficits due to political polarization.
- Long-term fiscal burdens and suboptimal spending patterns.

Policy Implications:

- Political polarization can exacerbate deficit bias.
- Institutional arrangements (fiscal rules, independent councils) can mitigate strategic debt incentives.

Delayed Stabilisation

Delayed Stabilization – An Overview

Delayed stabilization: Persistence of large fiscal deficits despite known economic costs.

- Deficits cause economic distortions and risk sovereign debt crises.
- Timely fiscal reforms (tax increases or spending cuts) can mitigate these issues.
- Yet, reforms often delayed due to political conflicts and bargaining among interest groups.
- Historical example: Post-WWI hyperinflation prolonged by disagreement over taxing capital vs. labor.
- Modern debates: Disagreements over spending cuts vs. tax hikes.
- Alesina and Drazen (1991) model illustrates delays due to political bargaining.

Model Setup – Bargaining Framework

- Two groups: Capitalists and workers (both risk-neutral).
- Must agree on fiscal reform and distribution of fixed tax burden $T > 0$.
- Without agreement: severe fiscal crisis; both groups receive zero payoff.
- With agreement:
  - Workers earn pre-tax income W (assume $W > T$).
  - Capitalists earn uncertain pre-tax income R, uniformly distributed $[A, B]$, known only to capitalists.
- Tax agreement involves capitalists paying X of tax T:
  - Capitalists' payoff: $R - X$
  - Workers' payoff: $W - T + X$
- Feasible agreements beneficial for both groups if $0 \leq X \leq A$.

Bargaining and Probability of Acceptance

- Workers propose a tax contribution X ("take-it-or-leave-it" offer).
  - If capitalists accept, reform implemented; otherwise, crisis occurs.
- Capitalists accept if $R \geq X$.
- Probability of acceptance given proposal X:

$$P(X) = \begin{cases} 1 & \text{if } X \leq A \\ \frac{B-X}{B-A} & \text{if } A < X < B \\ 0 & \text{if } X \geq B \end{cases}$$

- Workers uncertain about R but aware of acceptance probability $P(X)$.

Workers' Optimal Proposal

- Workers' expected payoff:

$$V(X) = P(X)(W - T + X) = \begin{cases} W - T + X & \text{if } X \leq A \\ \frac{(B-X)(W-T+X)}{B-A} & \text{if } A < X < B \\ 0 & \text{if } X \geq B \end{cases}$$

- Workers have two strategic options:
  1. Propose $X = A$ ensuring certain acceptance; payoff: $W - T + A$.
  2. Propose $X \in (A, B)$, accepting uncertainty for potentially higher payoff.
- Optimal proposal depends on maximizing expected payoff $V(X)$.

Optimal Decision and Equilibrium Probability

- First-order condition for optimal X when $A < X < B$:

$$V'(X) = \frac{B - (W - T) - 2X}{B - A} = 0 \quad \Rightarrow \quad X = \frac{B - (W - T)}{2}$$

- This is the interior optimal solution—the point at which marginal gain from higher taxes equals the marginal loss from lower probability of acceptance.
- Workers choose:

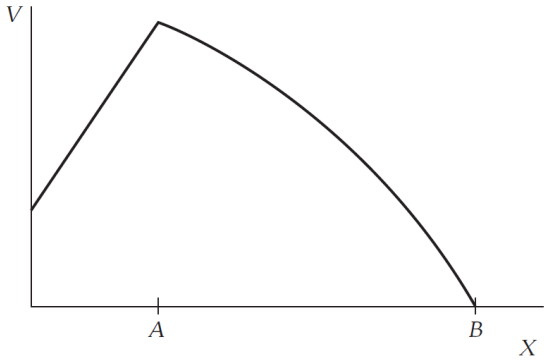
$$X^* = \begin{cases} A & \text{if } B - (W - T) - 2A \leq 0 \\ \frac{B-(W-T)}{2} & \text{if } B - (W - T) - 2A > 0 \end{cases}$$

- Equilibrium acceptance probability:

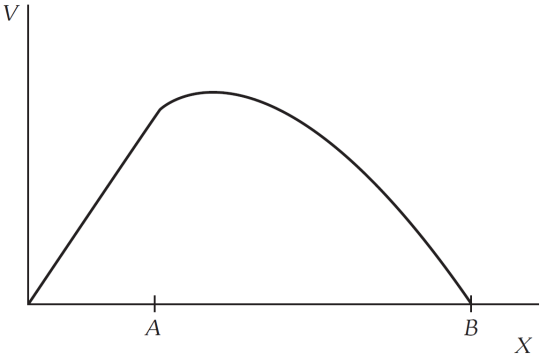
$$P(X^*) = \begin{cases} 1 & \text{if } B - (W - T) - 2A \leq 0 \\ \frac{B+(W-T)}{2(B-A)} & \text{if } B - (W - T) - 2A > 0 \end{cases}$$

- Higher uncertainty (larger $B - A$) reduces reform probability.

Examples: workers' expected payoff as a function of X



Workers pick $X^* = A$



Workers pick $X^* > A$

Implications of Delayed Stabilization Model

- Even mutually beneficial fiscal reforms can fail due to strategic bargaining.
- Workers may deliberately propose riskier terms seeking higher payoffs, increasing the likelihood of reform failure.
- Higher uncertainty in income distribution intensifies bargaining conflicts, delaying reforms.
- Countries with fragmented or weak governments (no clear decision-maker) are more prone to persistent deficits and delayed stabilization.
- Crisis situations (high costs of non-agreement) can sometimes incentivize rapid agreements, highlighting the complex role crises play in triggering necessary fiscal reforms.

Sovereign Debt Crises

Sovereign Default - Definition

→ **Narrow definition:** when the debtor violates the legal terms of the debt contract:

- Fail to pay interest or principal within the specified period.
- Breach some other contractual provision.
- This narrow definition overlooks situations in which the sovereign threatens to default and creditors respond by *voluntarily* revising the contract.

→ **Credit ratings agencies' definition:** when the sovereign breaks the debt contract or when the sovereign tenders a **distressed debt exchange** (i.e. an exchange offer of new debt with less favourable terms than the original debt).

- **Example:** Greece's debt restructuring did not trigger a narrow default: the government did not miss any payments and investors have not (successfully) alleged a technical breach. But the new terms offered by Greece were worse than those on the original debt, and ratings agencies conclude that a default had occurred.

Sovereign Debt Crises – An Overview

A government can't meet its debt obligations. This could happen either because it genuinely lacks sufficient funds (**insolvency**) or temporarily lacks access to sufficient liquidity (**illiquidity**).

Why might these crises occur suddenly?

- Crises often occur suddenly due to a rapid shift in investor sentiment.
- Even a small shift in economic fundamentals or investor beliefs can trigger a sudden refusal by investors to roll over existing debt.

Important questions addressed by the model:

1. Investor refusal to lend:
   - Investors refuse to purchase government bonds regardless of the interest offered due to a perceived high default risk, causing a liquidity-driven crisis.
2. Sudden crises from minor changes:
   - Small deteriorations in economic conditions (e.g., slightly higher debt or slightly lower expected revenue) can lead to large swings in investor sentiment. Thus, crises can happen quickly and unpredictably.

Model Setup – Rolling Over Government Debt

- Government has existing debt D maturing, with no immediate liquidity.
- To avoid default, the government must roll over debt by issuing new bonds to investors.
- Investors (risk-neutral) require minimum return $\bar{R} = 1 + \bar{r}$.
- Government offers interest rate $R = 1 + r$ (endogenous).
- Future tax revenue T is uncertain, described by cumulative distribution $F(T)$.
- If tax revenue $T \geq RD$, government repays debt.
- If $T < RD$, government defaults fully (all-or-nothing default).

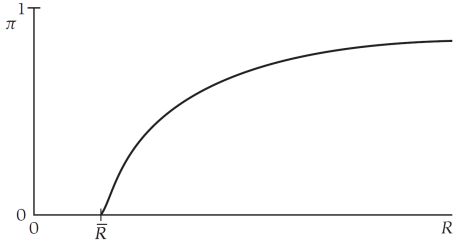
Investor Decision – Debt Demand Condition

- Investors' willingness to hold debt depends on the probability of default (π):

$$(1-\pi)R = \bar{R}$$

- When no default risk ($\pi = 0$), $R = \bar{R}$.
- As $\pi \to 1$, investors require $R \to \infty$.
- Debt demand curve (downward-sloping relationship between default probability and interest rate):

$$\pi = 1 - \frac{\bar{R}}{R}$$

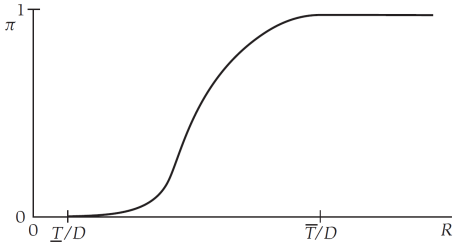


Probability of Default Condition

- Probability of default is the likelihood that future tax revenue T are insufficient to repay its debt obligations RD:

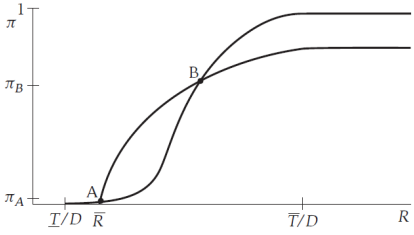
$$\pi = \Pr(T < RD) = F(RD)$$

- Typically, the distribution of tax revenue T is bell-shaped, yielding an S-shaped default probability curve:
  - At low interest rates, default probability is very low.
  - Probability sharply increases as R rises.
- Extreme outcomes:
  - $\pi = 0$ if RD is below minimum revenue (certain repayment).
  - $\pi = 1$ if RD exceeds maximum possible revenue (certain default).



Equilibrium – Multiple Equilibria

- Equilibrium defined by intersection of debt demand and default probability curves.
- Due to the curves' shapes, multiple equilibria possible:
  - Equilibrium A: Low default risk and interest rate close to safe rate $\bar{R}$.
  - Equilibrium B: High default risk and high interest rate.
- Additionally, there always exists a third equilibrium (certain default), where no investor is willing to buy government bonds at any finite interest rate.

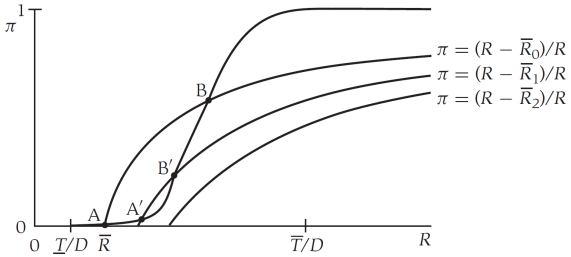


Stability of Equilibria and Investor Expectations

- Equilibrium stability determined by investors' expectations:
  - Equilibrium B (high risk) unstable: Small changes in expectations push outcomes toward extremes (either low-risk equilibrium or complete default).
  - Equilibrium A (low risk) and complete default equilibrium are stable.
- Self-fulfilling nature of crises:
  - Investor pessimism (expecting default) becomes self-fulfilling due to higher interest rates, increasing default risk.
  - Investor optimism maintains low default probabilities and interest rates.

Implications – Fundamentals and Crises

- Fundamentals strongly affect equilibrium outcomes:
  - Higher required safe return $\bar{R}$, increased debt D, or lower expected tax revenue shift equilibrium toward higher default risk.
- Small changes in fundamentals can trigger large shifts:
  - A modest rise in safe interest rate or debt levels can cause dramatic shifts from low-risk equilibrium to complete default.
  - Crises can thus arise suddenly and unexpectedly, even from minor economic changes.



Real-World Policy Implications

- Sovereign debt crises involve self-fulfilling dynamics:
  - Policy interventions that reduce investor uncertainty or improve credibility (e.g., clear fiscal commitments, credible monetary policies) can stabilize low-risk equilibria.
- National and international institutions (IMF, ECB, Central Banks) may act as lenders of last resort, stabilizing expectations.
- Preventive fiscal policy and maintaining sustainable debt levels critical to avoiding debt crises.

Summary and Conclusions

- Sovereign debt crises driven by interplay of economic fundamentals and investor expectations.
- Multiple equilibria highlight importance of investor beliefs:
  - Investor confidence reduces default likelihood.
  - Investor fear can cause self-fulfilling debt crises.
- Sudden shifts to default equilibrium possible with minor fundamental changes.
- Robust fiscal frameworks and credible economic policy are key to crisis prevention.