MSc Financial Economics (Macro)Economics Problem Set -

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Question 1. *Getting to know the Shapiro and Stiglitz (1984) model [Romer (2018) 11.3]*

Describe how each of the following affects equilibrium employment and the wage in the Shapiro Stiglitz model:

(a) An increase in workers' discount rate, ρ.
(b) An increase in the job breakup rate, b.
(c) A positive multiplicative shock to the production function (that is, suppose the production function is $AF(L)$, and consider an increase in A).
(d) An increase in the size of the labour force, $\bar{L}$.

Provide economic intuition for each result.

Question 2. *To let shirk or not to let shirk?*

This question asks about a highly simplified version of the Shapiro-Stiglitz shirking model covered in the lecture. We are interested in whether it makes sense for a firm to monitor its workers and pay wages sufficiently high to prevent shirking. The problem was trivial in the lecture because shirking workers were assumed to be completely unproductive. To make the problem interesting we therefore assume that even workers who shirk and exert zero effort have some productivity. In particular, we assume that shirking workers have productivity 1 and non-shirking workers have productivity $x > 1$, so the firm receives revenue 1 per worker if the worker shirks and revenue x if the worker does not shirk. Monitoring is assumed to have unit cost, so the expected cost of monitoring at the rate q is simply q. The expected profit (per worker) of a firm monitoring at the rate q and paying wage w is therefore:

$$\pi = R - w - q \tag{1}$$

where $R = x$ if the worker exerts effort $\bar{e}$ and $R = 1$ if the worker shirks and exerts no effort.

Building on the results we derived in the lecture, assume that to induce effort $\bar{e}$, the firm must pay the no-shirking wage:

$$w = \bar{e} + \mu\frac{\bar{e}}{q} \tag{2}$$

i.e. compensate the cost of effort $\bar{e}$ plus pay a premium of $\mu\frac{\bar{e}}{q}$. μ here is a function of workers' preferences and aggregate macroeconomic conditions that the firm takes as given when setting its own wage and the rate of monitoring q. Assume also that a worker who is shirking and exerting zero effort is willing to take up the job at any non-negative wage.

(a) Suppose the firm decides not to bother satisfying the non-shirking condition (2) and simply lives with shirking workers. What monitoring probability will the firm set if they maximise expected profit? What wage will it pay? What will be its profits?

(b) Suppose the firm now decides to set the monitoring probability and wage to induce its workers not to shirk. Set up an optimisation problem for the firm to determine the optimal monitoring rate q^* and optimal wage w^* to prevent shirking. Discuss how and why these variables depend on the primitives of the model. Based on the lecture, what factors can affect μ, and how do changes in μ affect the firm's choices?

(c) Show that there will be no shirking in this economy iff

$$x - \bar{e} - 2\sqrt{\mu\bar{e}} > 1 \tag{3}$$

Interpret.

Question 3. *The fair wage-effort hypothesis (Akerlof and Yellen 1990) [Romer (2018) 11.5]*

Preamble: This problem introduces you to the fair wage-effort hypothesis, which is another efficiency wage theory, and a nice alternative to the Shapiro-Stiglitz model covered in the lecture. The maths here is very simple, but if you get stuck, there will be solutions next week, with a little bonus for those who get it right.

Suppose there are a large number of firms, N, each with profits given by $F(eL) - wL$, $F'(\cdot) > 0$, $F''(\cdot) < 0$. L is the number of workers the firm hires, w is the wage it pays, and e is workers' effort. Effort is given by $e = \min[w/w^*, 1]$, where w^* is the "fair wage"; that is, if workers are paid less than the fair wage, they reduce their effort in proportion to the shortfall. Akerlof and Yellen motivate the fair wage-effort hypothesis by an observation that human behaviour is strongly driven by the sense of fairness and equity: when employees do not get what they think they deserve, they try to get even at their employers. This theory has strong grounding in psychology, sociology, and is considered obvious in HR textbooks. You are invited to investigate its macroeconomic implications.

Assume that there are $\bar{L}$ workers who are willing to work at any positive wage.

(a) If a firm can hire workers at any wage, what value (or range of values) of w minimizes the cost per unit of effective labour, w/e? For the remainder of the problem, assume that if the firm is indifferent over a range of possible wages, it pays the highest value in this range.

(b) Suppose w^* is given by $w^* = \bar{w} + a - bu$, where u is the unemployment rate and $\bar{w}$ is the average wage paid by the firms in the economy. Assume $b > 0$ and $a/b < 1$.

   (i) Given your answer to part (a) (and the assumption about what firms pay in cases of indifference), what wage does the representative firm pay if it can choose w freely (taking $\bar{w}$ and u as given)?

   (ii) Under what conditions does the equilibrium involve positive unemployment and no constraints on firms' choice of w? (Hint: In this case, equilibrium requires that the representative firm, taking $\bar{w}$ as given, wishes to pay $\bar{w}$.) What is the unemployment rate in this case?

   (iii) Under what conditions is there full employment?

(c) Suppose there are two types of workers: high-productivity and low-productivity. The representative firm's production function is modified to be $F(Ae_1L_1 + e_2L_2)$, $A > 1$, where L_1 and L_2 are the numbers of high-productivity and low-productivity workers the firm hires.

Assume that $e_i = \min[w_i/w_i^*, 1]$, where w_i^* is the fair wage for type-i workers. w_i^* is given by $w_i^* = [(\bar{w}_1 + \bar{w}_2)/2] - bu_i$, where $b > 0$, $\bar{w}_i$ is the average wage paid to workers of type i , and u_i is their unemployment rate.

Therefore, the fair wage for each type partly depends on the wages received by other members of the workforce.[1] Finally, assume there are $\bar{L}$ workers of each type.

(i) Explain why, given your answer to part (a) (and the assumption about what firms pay in cases of indifference), neither type of worker will be paid less than the fair wage for that type.

(ii) Explain why w_1 will exceed w_2 by a factor of A.

(iii) In equilibrium, is there unemployment among high-productivity workers? Explain. (Hint: If u_1 is positive, firms are unconstrained in their choice of w_1.)

(iv) In equilibrium, is there unemployment among low-productivity workers? Explain.

Question 4. *Getting to know the Search and Matching model*

Consider the DIAMOND-MORTENSEN-PISSARIDES (Diamond 1982; Mortensen and Pissarides 1994; Pissarides 2000) search and matching model studied in the lecture.

Describe how each of the following affects steady-state (un)employment and output:

(a) An increase in unemployment benefits, b.
(b) An decrease in the matching efficiency, e.
(c) An increase in the productivity, z.

Provide economic intuition with analitical mechanism for each result.

Question 5. *The Beveridge curve in recent years*

Figure 1 plots vacancies against unemployment in the US for December 2000 - December 2024. However, when I have split the Beveridge-curve data into multiple time periods and fit separate trend lines for each, as shown in Figure

[1] We assume a simple average of the mean wages in the two groups for simplicity only, and this can be relaxed.

2, I get different slopes for the trend lines.Why do the trend lines have different slopes? What economic factors might explain these varying slopes?

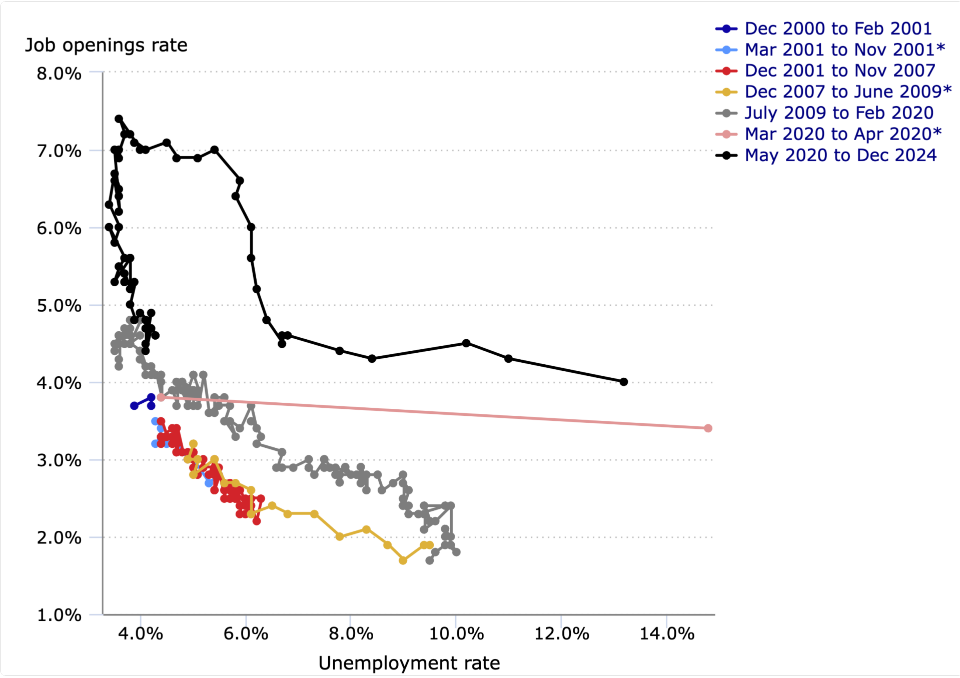
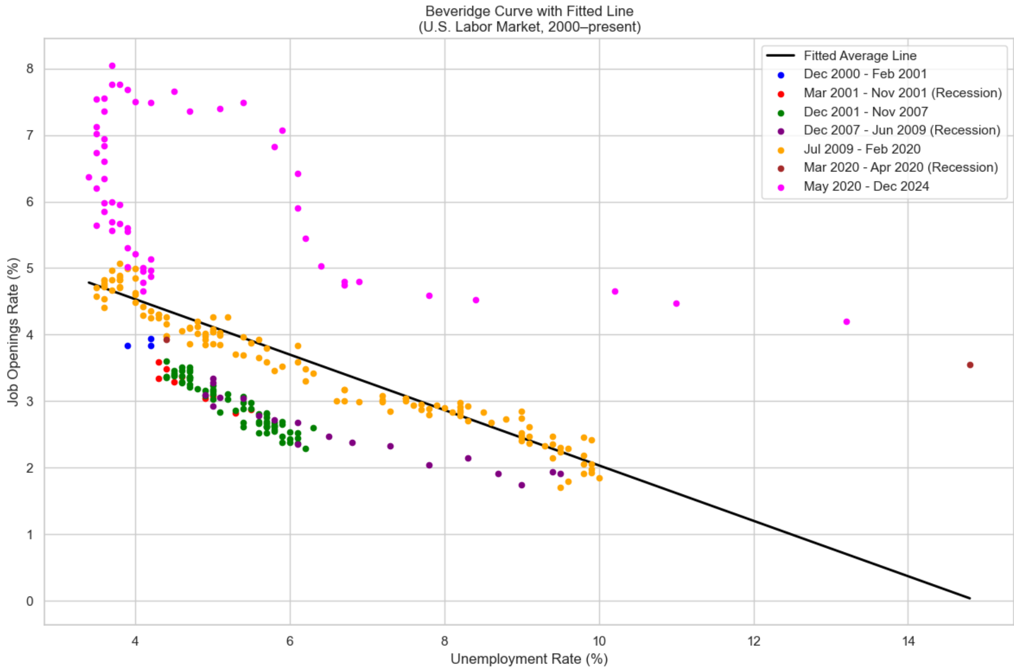
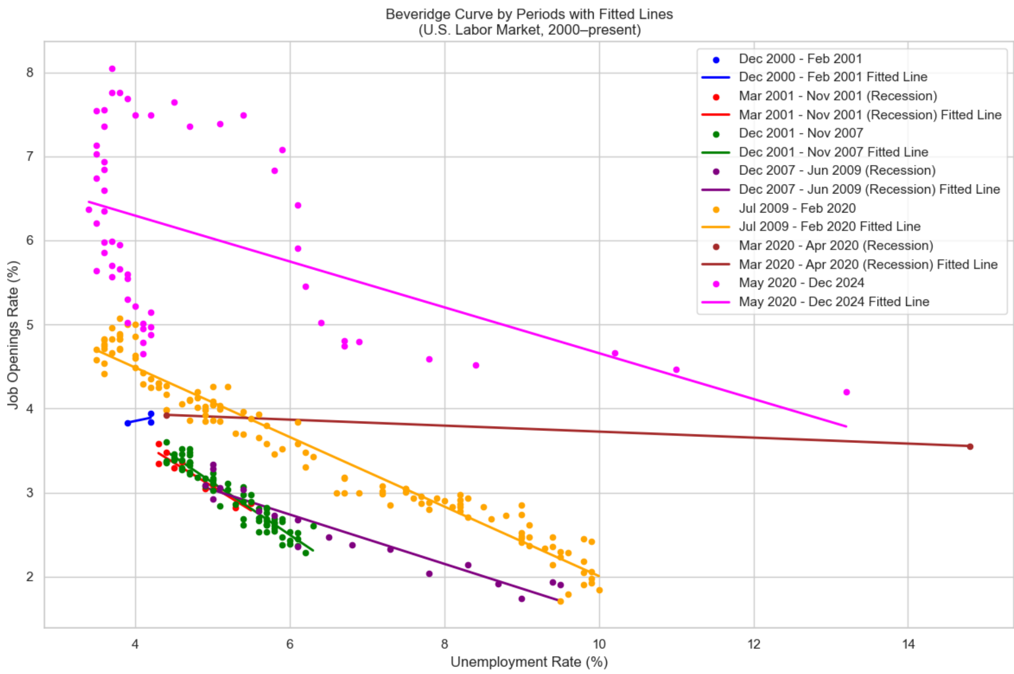


FIGURE 1. The Beveridge curve in the US, December 2000 - December 202d

* *Represents recession, as determined by NBER*: Source: US Bureau of labour Statistics.



(A) Fitted line for the period 2000-2024



(B) Different fitted lines for different periods

FIGURE 2. Beveridge Curve and Fitted Lines

Notes & Source: Data: US Bureau of labour Statistics. Calculations: Fatih Kansoy.

Question 6. *Financial market imperfections* Machines are used to produce a single output good in a two period economy. The stock of machines in period

$t = \{1, 2\}$ is denoted K_t and produces output goods AK_t in period t with $A > 1$. The stock of machines in period 1, K_1, is given and machines never depreciate. At the end of period 1 a one-for-one technology is available which converts output into machines, so the representative agent chooses how to allocate output AK_1 between consumption C_1 in period 1 and an investment to increase the stock of machines in period 2. The utility of the representative agent is $\ln(C_1) + \ln(C_2)$, so there is no discounting. The representative agent has no initial debt, but can borrow (from abroad) at the rate $R \in (1, A)$ in period 1 to finance either consumption in period 1 or additional investment in machines for period 2. Denote the amount borrowed by the representative agent in period 1 as B, to be repaid in period 2 with interest. Crucially, there is a financial market imperfection in the economy: the representative agent cannot borrow more than a fraction $\psi < 1$ of the quantity of their machines in the second period. That is, the machines serve as collateral for the loan.

(a) Set up the maximisation problem of the representative agent.

(b) Derive the agent's optimality condition(s). How does the Euler equation for consumption look in this case? Interpret.
Hint: You could solve this problem by setting up the Lagrangian and writing down the Kuhn-Tucker first-order conditions. Alternatively, argue why all constraints faced by the agent, including the constraint on borrowing, must bind at the optimum, and then solve the problem by substituting the constraints into the objective function.

(c) Solve for C_1, C_2, K_2 and B as functions of exogenous parameters A, ψ, R and K_1.

(d) Describe the effects of an increase in ψ, which can be interpreted as an increase in the degree of financial openness of the economy

Question 7. *Bankers cannot abscond with government money* This question builds on the Gertler-Kiyotaki model we studied in the lecture by incorporating the government as an additional agent. A representative household has an endowment y in period 1 of a two-period economy. They have preferences $\sqrt{c_1} + \beta\sqrt{c_2}$ over consumption in the two periods, where β is the discount factor. The government imposes a lump sum tax on the household in period 1. The government then deposits the tax revenue T at the bank in period 1 and distributes the return RT as a lump sum tax rebate to households in period 2. The representative bank takes in deposits from both the consumer (d) and the government (T) in period 1, adds its own net wealth N, and invests the total with entrepreneurs for return R^k. The bank's revenue in period 2 is therefore $R^k(d + T + N)$. The banker can default in period 2 before repaying consumer deposits, but if he does so he can only abscond with a fraction θ of the funds he owes to households and owns himself. He cannot abscond with any of the government deposits.

(a) Define and solve the utility maximisation problem of a representative household, assuming they take as given the deposit rate R, the lump-sum tax T, and the banker's profit π. How do lump-sum taxes affect deposits d and the household's consumption choices? Why do you think this is the case?

(b) Write down a sufficient condition that stops the banker from defaulting. Assuming this is satisfied in equilibrium,[2] define the profit maximisation problem of the bank, and discuss its implications. How do lump-sum taxes affect the behaviour of the banker?

(c) What is the socially optimal (first-best) consumption allocation in the absence of financial frictions?

(d) Discuss whether the government can use lump sum taxation to implement the first-best consumption allocation in part (c). How realistic do you think the underlying assumptions are?

Question 8. *Modifying assumptions in the Diamond-Dybvig model [Romer (2018), pp. 382]* Consider the Diamond Dybvig model we studied in the lecture, but suppose that $\rho R < 1$.

(a) In this case, what are c_1^{a*} and c_2^{b*}? Is c_2^{b*} still larger than c_1^{a*}?

(b) Suppose the bank offers the contract described in the lecture: anyone who deposits one unit in period 0 can withdraw c_1^{a*} in period 1, subject to the availability of funds, with any assets remaining in period 2 divided equally among the depositors who did not withdraw in period 1. Explain why it is not an equilibrium for the type-a's to withdraw in period 1 and the type-b's to withdraw in period 2.

(c) Is there some other arrangement the bank can offer that improves on the autarky outcome?

Question 9. *Deposit insurance in the Diamond-Dybvig model [Romer 10.12]* Consider deposit insurance in the Diamond Dybvig model as studied in the lecture.

[2] In the lecture, we discussed the precise reasons for why, without loss of generality, we can restrict attention only to the equilibria in which banks never default.

(a) Suppose fraction $\phi > \theta$ of depositors withdraw in period 1. How large a (lump-sum) tax must the government levy on each agent withdrawing in period 1 to be able to increase consumption of those agents who wait to withdraw until period 2 to c_2^{b*}? Explain why your answer should simplify to zero when $\phi = \theta$, and check that it does.

For concreteness, suppose that in period 1 the government collects taxes *after* agents withdraw funds from the bank. Since the projects are already liquidated at that point, the government *does not* earn return R on the collected resources.

(b) Suppose the tax is marginally less than the amount you found in part (a). Would the type-b's still prefer to wait until period 2 rather than try to withdraw in period 1?

Question 10. *Strategic debt accumulation when there is initial debt [Romer 13.7]* Consider the Tabellini-Alesina (1990) model of strategic debt accumulation we studied in the lecture. Suppose that there is some initial level of debt, D_0. How, if at all, does D_0 affect the deficit in period 1?

Question 11. *Comparative statics in the model of delayed stabilization [Romer 13.11]* Consider the Alesina-Drazen (1991) model of delayed stabilization we studied in the lecture. Describe how, if at all, each of the following developments affects workers' proposal and the probability of reform:

(a) A fall in T.
(b) A rise in B.
(c) An equal rise in A and B.

Question 12. *Crises and reforms [Romer 13.12]* Consider the Alesina-Drazen (1991) model of delayed stabilization we studied in the lecture. Suppose, however, that if there is no reform, workers and capitalists both receive payoffs of $-C$ rather than 0, where $C \geq 0$. In other words, a failure to agree leads to a deeper crisis.

(a) Find expressions analogous to (17) and (18) in the lecture slides for workers' proposal and the probability of reform.
(b) Define social welfare as the sum of the expected payoffs of workers and capitalists. Show that an increase in C can raise this measure of social welfare.

Question 13. *A model of sovereign debt crises [Romer 13.16]* Consider the model of sovereign debt crises we studied in the lecture. Suppose T is distributed uniformly on some interval $[\mu - X, \mu + X]$, where $X > 0$ and $\mu - X \geq 0$. Describe how, if at all, each of the following developments affects the debt demand and default probability curves in the (R, π) space, as well as determination of R and π in equilibrium:

(a) A rise in μ.
(b) A fall in X.

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