Unemployment

MSc Financial Economics

Fatih Kansoy

Hilary Term

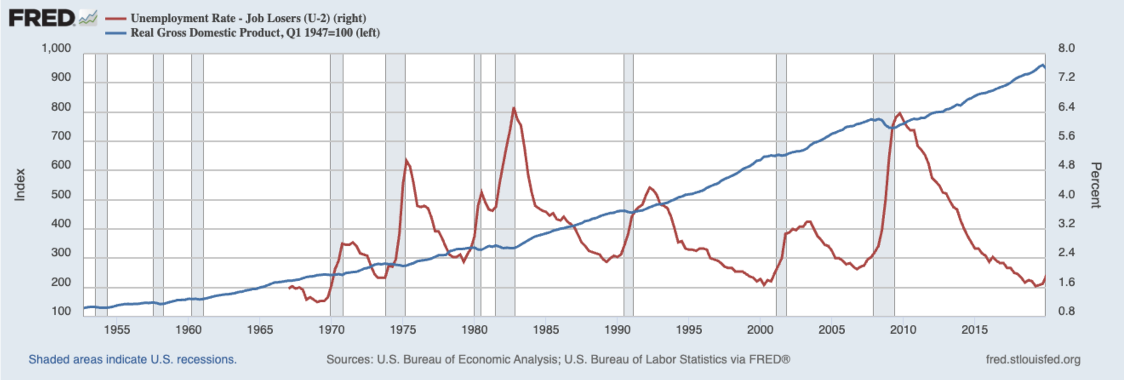
University of Oxford

Outline

Questions and Terminology

Unemployment Despite Growth

Why does unemployment persist even in an economy with constant growth and rising labour demand?



Was Henry Ford right?

'GOLD RUSH' IS STARTED BY FORD'S $5 OFFER

Thousands of Men Seek Employment in Detroit Factory.

Will Distribute $10,000,000 in Semi-Monthly Bonuses.

No Employe to Receive Less Than Five Dollars a Day.

(TIMES-STAR SPECIAL DISPATCH.)
DETROIT, Mich., January 7.—Henry Ford in an interview to-

Can firms always find workers whenever they need them?

HARVARD BUSINESS REVIEW

Latest Magazine Topics Podcasts Store The Big Idea Da

Business Education

The U.S. Education System Isn't Giving Students What Employers Need

by Michael Hansen

May 18, 2021

Chris Ryan/Getty Images

- The estimated number of vacancies was 819,000 in the UK in November 2024 to January 2025; this is a decrease of 9,000, or 1.1%, from August to October 2024.
- The number of unemployed people in the UK was approximately 1.56 million in the period of October to December 2024.

Unemployment

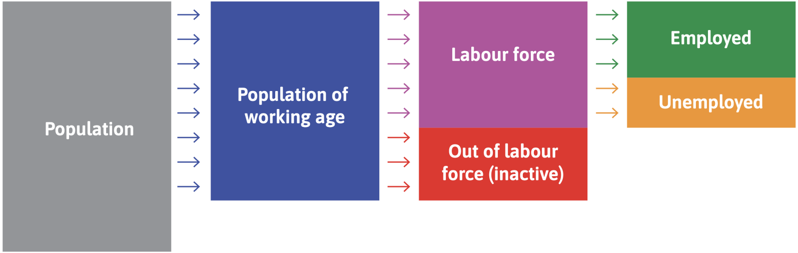
According to the standardised definition of *the International labour Organization* (**ILO**):

- **Employment** is the number of people who have a job.

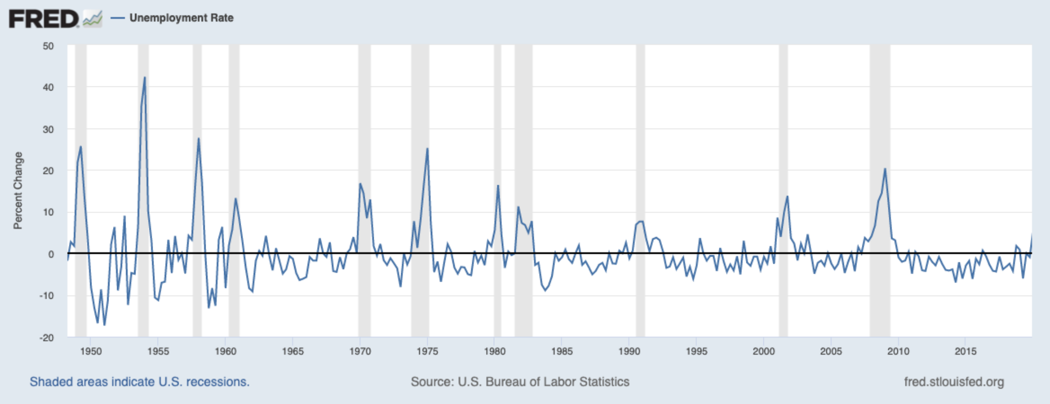
Unemployment

According to the standardised definition of *the International labour Organization* (**ILO**):

- **Employment** is the number of people who have a job.
- **Unemployment** is the number of people who do not have a job but are looking for one.
  - an unemployed person is a person aged 15 or over:
  - without a job during a given week;
  - available to start a job within the next two weeks;
  - actively having sought employment at some time during the past four weeks or having already found a job that starts within the next three months.

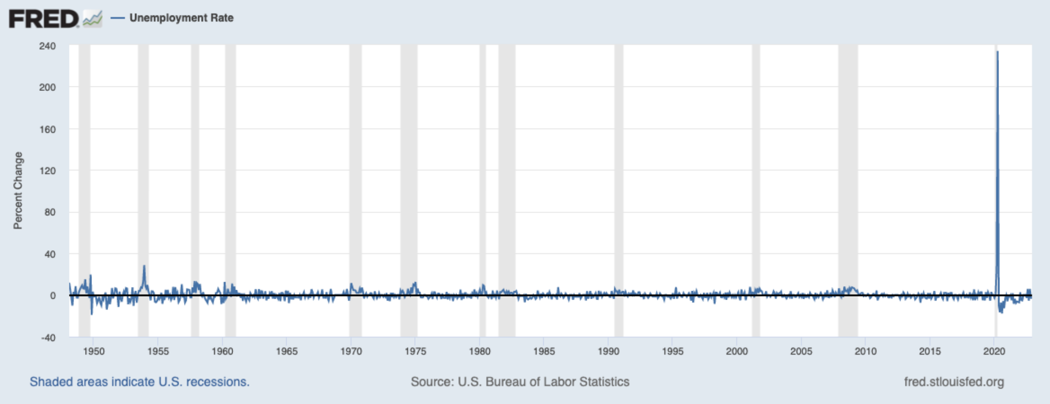


US Unemployment Rate - 1950-2019



Source. FRED. Series UNRATE. .

US Unemployment Rate - 1950-2022



Source. FRED. Series UNRATE. .

Why it matters?

Why Do Economists Care about Unemployment?

1. Because of its direct effect on the welfare of the unemployed, especially those remaining unemployed for long periods of time.

2. It is a signal that the economy is not using its human resources efficiently.

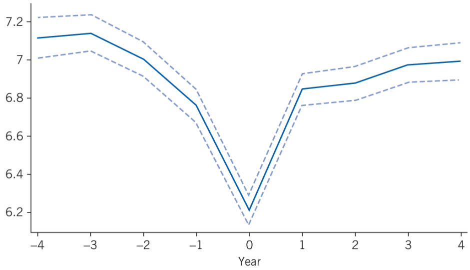
When unemployment is high:

- Many workers who want to work do not find jobs; the economy is clearly not using its human resources efficiently.

What about when unemployment is low?

- An economy in which unemployment is very low may be overusing its resources and run into labour shortages (particularly for skilled workers).

Unemployment and Happiness



Source. Rainer Winkelmann, IZA, 2014. .

Source. Rainer Winkelmann, IZA, 2014. .

To give you a sense of scale, other studies suggest that this decrease in happiness is close to the decrease triggered by a divorce or a separation.

Today's Agenda

- We will consider three highly influential models of the labour market:

  1. **Efficiency wage theory**
  2. **Shapiro-Stiglitz model,** which formally explores the deeper reasons for efficiency wages.
  3. **Search and matching model.**

- The first two are examples of the **traditional approach** of modelling the labour market within the standard supply and demand framework.

- The last one is an example of the **modern approach.**

  ⇒ Focus on the costly process of job search and recruitment.

Efficiency Wage Models

Efficiency wage theories: overview

- The key idea of efficiency wage theories is that there are benefits of paying higher wages to employees.
- Among suggested reasons, the following received the most attention:
  1. Better **nourishment**, and thus productivity.
  2. **Incentive to exert high effort** when firms can't monitor workers perfectly, as in the Shapiro-Stiglitz (1984) model – later today.
  3. Higher wages can **attract workers of higher ability**.
  4. **The fair wage-effort hypothesis** due to Akerlof and Yellen (1990): high wage can build loyalty and hence induce effort – problem set .

     $\Rightarrow$ extensive evidence that workers' effort is affected by such feelings as anger, jealousy, and gratitude.
- We begin with a simple efficiency wage model due to Solow (1979).

Setup

- No capital for simplicity, labour is the only factor of production.
- There is a large number N of firms and a representative firms maximizes profits:

$$\pi = Y - wL, \tag{1}$$

- where Y is the firm's output, w is the wage that it pays, and L is the amount of labour it hires.
- Output depends on the number of workers and on their effort; $Y = F(eL)$.
- Thus the representative firm's output:

$$\pi = F(eL) - wL, \quad F'(\bullet) > 0,\ F''(\bullet) < 0, \tag{2}$$

where e is workers' **effort**, so eL is **effective labour**.

Setup

- Assume effort $\boldsymbol{e}$ is an increasing function of the wage:

$$\boldsymbol{e = e(w), \quad e'(\bullet) > 0} \tag{3}$$

- For now, we are interested in the implications, and not precise reasons.

- There are $\boldsymbol{\bar{L}}$ workers, each supplying 1 unit of labour *inelastically*

  $\Rightarrow$ i.e prepared to work at any wage.

The firm's problem

- A representative firm solves

$$\max_{L,w} \; \boldsymbol{F(e(w)L) - wL}. \tag{4}$$

- There can be two cases:

  1. There are unemployed workers, and the firm can choose its wage freely.
  2. There is zero unemployment, so the firm must pay at least the wage paid by other firms to attract any workers.

- The first-order condition (FOC) with respect to $\boldsymbol{L}$ yields:

$$\boldsymbol{F'(e(w)L) = \frac{w}{e(w)}} \tag{5}$$

- This means the marginal product of *effective* labour equals its unit cost, $\boldsymbol{\frac{w}{e(w)}}$
- *Note:* When a firm hires a worker, it gets $\boldsymbol{e(w)}$ units of effective labour.

The efficiency wage

- When the firm is unconstrained and sets wage freely, taking FOC w.r.t. $\boldsymbol{w}$ from (4) yields

$$\boldsymbol{F'(e(w)L)Le'(w) - L = 0}. \tag{6}$$

- Substitute for $\boldsymbol{F'(e(w)L)}$ from (5) and rearrange to get:

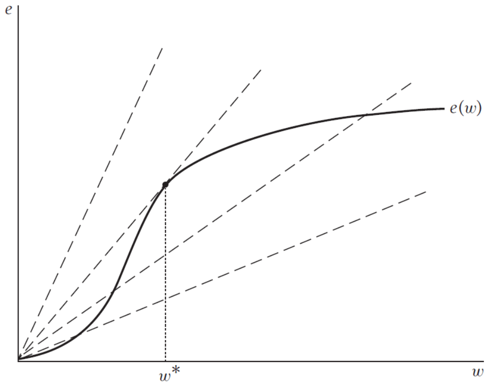
$$\boldsymbol{\frac{we'(w)}{e(w)} = 1} \tag{7}$$

- i.e. at the optimum, elasticity of effort w.r.t. wage is $\boldsymbol{1}$.

The efficiency wage

$$\frac{we'(w)}{e(w)} = 1 \qquad (8)$$

- The firm wants to hire effective labour, eL, as cheaply as possible.
- (8) defines the **efficiency wage** that **minimises the unit cost** of effective labour, i.e. solves $\min_w w/e(w)$
- Put differently, optimal w **maximises effort per dollar spent**, $e(w)/w$



Equilibrium

- Let $\boldsymbol{L}^*$ and $\boldsymbol{w}^*$ denote the values that satisfy conditions (5) and (7).
- Since all firms are identical, the total labour demand at $\boldsymbol{w}^*$ is $\boldsymbol{NL}^*$.

1. If $\boldsymbol{NL}^* < \bar{\boldsymbol{L}}$, then there is **positive unemployment** in equilibrium:
    - Firms are free to set wages, so the equilibrium wage is simply $\boldsymbol{w}^*$.
    - At this wage, employment is indeed given by the labour demand, $\boldsymbol{NL}^*$.
    - $\bar{\boldsymbol{L}} - \boldsymbol{NL}^*$ workers are unemployed.
2. If $\boldsymbol{NL}^* > \bar{\boldsymbol{L}}$, there is **full employment** in equilibrium:
    - At $\boldsymbol{w}^*$, labour demand would exceed supply,
    - so the wage is bid up above $\boldsymbol{w}^*$ in equilibrium until $\boldsymbol{NL}(\boldsymbol{w}) = \bar{\boldsymbol{L}}$.
    - Firms are constrained, and unable to reduce the wage to $\boldsymbol{w}^*$.

Implications

1. The model implies the possibility of **involuntary unemployment:**
   - When $\boldsymbol{NL^*} < \bar{\boldsymbol{L}}$ and $\boldsymbol{w} = \boldsymbol{w}^*$, there are workers who want to work at the prevailing wage, yet can't find employment.
   - The wage does not fall to equilibrate labour supply and demand, since it reflects efficiency considerations of maximizing workers' effort.
2. The model also sheds light on the labour market dynamics over the business cycle:
   - There is no reason for the firms to adjust real wages in response to, say, a negative demand or productivity shock.
   - The model thus predicts that shifts in labour demand will lead to large movements in employment, with little changes in the wages.

A Practical Caveat: Compensation Design

- So far we assume compensation is a simple wage $\boldsymbol{w}$.
- In reality, firms may use richer contracts to obtain effort at lower cost:
  - in-kind benefits (e.g. meals, transport),
  - bonuses and deferred pay,
  - performance-contingent contracts.
- Why this matters for us:
  - Efficiency-wage logic survives, but **measured wage rigidity** may overstate total compensation rigidity.
  - Empirically, we should interpret the model as a mechanism, not a literal description of every pay scheme.

Case study: Henry Ford and His Experiment!

https://macroeconomics.info/casestudies/w6_cs

- In 1914, Henry Ford instituted a $5 a day minimum wage for his workers.
- This was double the going wages at the time!
- Ford himself:

  "There was... no charity in any way involved. ... The payment of $5 a day for an eight hour day was one of the finest cost cutting moves we ever made"


'GOLD RUSH' IS STARTED BY FORD'S $5 OFFER

Thousands of Men Seek Employment in Detroit Factory.

Will Distribute $10,000,000 in Semi-Monthly Bonuses.

No Employe to Receive Less Than Five Dollars a Day.

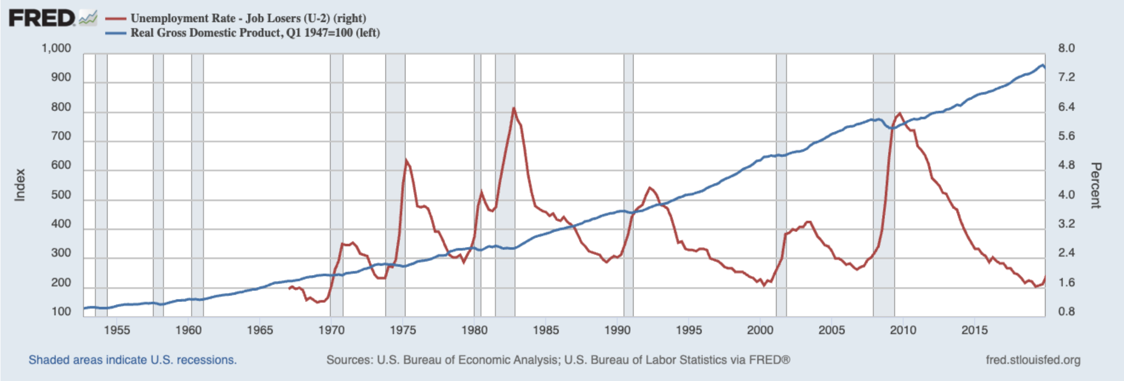
(TIMES-STAR SPECIAL DISPATCH.)

DETROIT, Mich., January 7.—Henry Ford in an interview to-

BUT

- Unchanged Real Wage Over Time
  - In the model, as labour demand grows, the real wage remains constant for a long stretch.
  - Eventually, as unemployment falls to zero, any further increase in labour demand must raise the real wage.
- Mismatch with Observed Unemployment Trends
  - In reality, unemployment does not trend steadily toward zero.
  - The model's prediction of a declining unemployment rate over time clashes with long-run data, which shows no clear trend in unemployment.

Unemployment and GDP



Empirical Puzzle

- Short-Run vs. Long-Run Behavior
  - Short run: labour-demand shifts primarily affect employment rather than the real wage.
  - Long run: Those same shifts should eventually move the real wage more than employment, but we do not observe that pattern.
- Unanswered Questions
  - Why do real wages not remain constant as demand grows over long periods?
  - How to reconcile no clear trend in unemployment with a model implying a downward trend?
  - The efficiency wage framework alone does not resolve these issues, indicating a need for additional mechanisms or explanations.

A More General Efficiency-Wage Framework

Why Extend the Basic Model?

- In the simplest efficiency-wage model, effort depends only on a single firm's wage.
- But in reality, workers also compare that firm's wage to what other firms pay, and they weigh the risk of being fired when unemployment is high or low.
- The basic efficiency-wage idea now includes:
  1. The wage the firm itself pays, $\boldsymbol{w}$.
  2. The wage paid by other firms, $\boldsymbol{w_a}$.
  3. The unemployment rate, $\boldsymbol{u}$.

New Effort Function

$$\boldsymbol{e = e(w, w_a, u)}$$

A More General Efficiency-Wage Framework

Effort Function

$$e = e(w, w_a, u),$$

subject to

$$\frac{\partial e}{\partial w} > 0, \quad \frac{\partial e}{\partial w_a} < 0, \quad \frac{\partial e}{\partial u} > 0.$$

Intuition:

1. Higher own wage (w) raises a worker's effort.
2. Higher outside wage (w_a) lowers effort (because alternative jobs look better).
3. Higher unemployment (u) raises effort (fear of job loss).

The Representative Firm's Problem

Production Side: Let the firm's production be $F(e(w, w_a, u)\,L)$

Profit Maximization: Choose w to balance wage costs against effort gains.
When the firm is price-taking in both product and labour markets, the key first-order conditions can be rearranged to:

$$F'(e(w, w_a, u)\,L) = \frac{w}{e(w, w_a, u)},$$

$$w\,\frac{\partial e/\partial w}{e(w, w_a, u)} = 1.$$

Interpretation:

- The marginal product of effective labour, F', must equal the ratio of the wage to effort.
- The elasticity of effort w.r.t. the firm's own wage is exactly 1 in equilibrium.

A Specific Example (Summers, 1988)

$$e = \begin{cases} \left(\frac{w - x}{x}\right)^{\beta} & \text{if } w > x, \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

For detailed explanation go to https://macroeconomics.info/.

where $x = (1 - bu)w_a$ which is a measure of how attractive the outside option is to the worker.

Parameters:

- $0 < \beta < 1$.
- $b > 0$ measures how strongly unemployment, u, affects a worker's perceived outside options.

Mechanics:

- If $w \leq x$, workers exert no effort.
- For $w > x$, effort increases less than proportionally with $(w - x)$

Solving for Equilibrium Wages and Unemployment

1. Start from the firm's wage FOC, (7):

$$\frac{w\,e'(w)}{e(w)} = 1, \qquad e(w) = \left(\frac{w-x}{x}\right)^{\beta} \text{ for } w > x.$$

   This gives

$$e'(w) = \beta\left(\frac{w-x}{x}\right)^{\beta-1}\frac{1}{x} \quad \Rightarrow \quad \frac{w\,e'(w)}{e(w)} = \frac{\beta w}{w-x} = 1.$$

2. Equilibrium wage:

$$\beta w = w - x \;\Rightarrow\; w = \frac{x}{1-\beta} \;=\; \frac{1-bu}{1-\beta}\,w_a.$$

3. Consistency (firm chooses the prevailing wage): impose $w = w_a$:

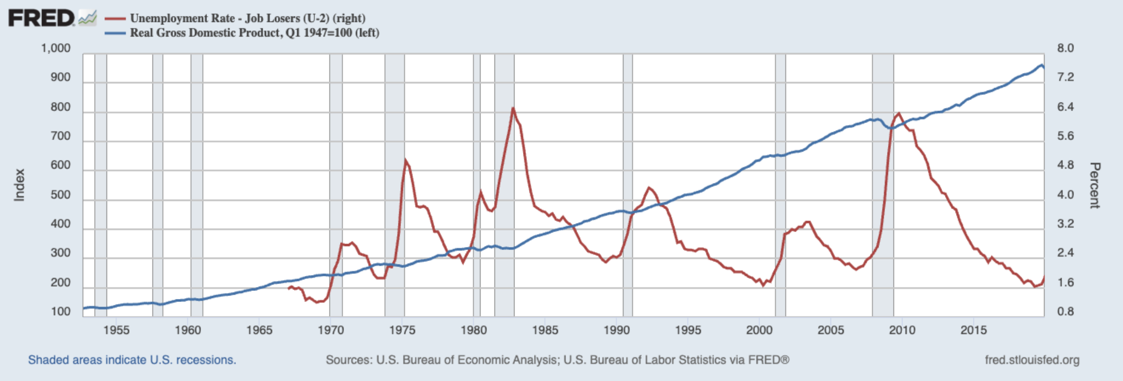
$$(1-\beta)\,w_a \;=\; (1-b\,u)\,w_a \quad\Longrightarrow\quad u \;=\; \frac{\beta}{b}. \tag{10}$$

   Interpretation: u_{EQ} depends only on β and b, so in this setup it is independent of long-run growth.

Unemployment and GDP

Unemployment Rate is independent of the long-run growth rate

$$u = \frac{\beta}{b}$$



Shapiro-Stiglitz Model

Shapiro-Stiglitz (1984) model: an overview

- So far we have simply been assuming that workers' effort is an increasing function of the wage in (3).
- In the highly influential paper, Shapiro and Stiglitz (1984) provide a microeconomic rationale for this assumption.
- **Idea:** if firms have a limited ability to monitor their workers, they are forced to provide them with enough incentives to exert high effort.
- In the model such incentive arises from the risk of being fired and loosing a well-paid job if not working hard (shirking).
- Not only does the model provide a logical justification for the efficiency wage, but it also does it in a spectacularly elegant fashion.

Assumptions of the Model

- The economy has:
  - A large number of workers $\bar{L}$ and firms N.
  - Workers maximize **lifetime utility**:

$$U = \int_0^{\infty} e^{-\rho t} u_t \, dt, \quad \rho > 0 \tag{11}$$

  where u_t is the instantaneous utility at time t and ρ is the discount rate.

- Instantaneous utility:

$$u_t = \begin{cases} w_t - e_t, & \text{if employed, effort exerted} \\ 0, & \text{if unemployed} \end{cases} \tag{12}$$

- Effort e_t has two levels: $e_t = 0$ (shirking) or $e_t = \bar{e} > 0$ (effort).

Worker States

$$u_t = \begin{cases} w_t - e_t, & \text{if employed, effort exerted} \\ 0, & \text{if unemployed} \end{cases}$$

At any moment in time, a worker can be in one of three states:

- Employed and exerting effort E and getting utility $w_t - \bar{e}$
- Employed and shirking S and getting utility w_t
- Unemployed U and getting utility 0

Transitions between states follow simple Poisson processes.

Job Ends: Exogenous Reasons - b - From E to U

- Jobs end randomly (exogenously) at a rate b and $b > 0$.
- The probability of the job surviving at a some later time t is:

$$P(t) = e^{-b(t-t_0)} \tag{13}$$

For detailed explanation go to https://macroeconomics.info/.

- The equation (13) states that the probability of the job surviving decays exponentially, with faster decay for higher b.
- More importantly, (13) implies that $P(t+\tau)/P(t) = e^{-b\tau}$, which is the probability that the worker is still employed time τ later.
- Thus, the job surviving is independent of t, the length of employment.
- Thus, the risk of job loss is memoryless, or job breakups follow a Poisson process.

Job Ends: Shirking Detection - q - From S to U

- q is the probability per unit of time that a shirker is detected.
- q is exogenous and independent of job separations b.
- Firms detect shirkers and fire them.
- The probability that a shirker is still employed time τ later is $e^{-q\tau} \times e^{-b\tau}$ the probability that the job survives time τ later.
- Combined probability of survival and no shirking detection:

$$P(t) = e^{-b(\tau)} \cdot e^{-q(\tau)} = e^{-(b+q)(\tau)} \tag{14}$$

$q \to \infty$? https://youtu.be/dHcxTmU6atk.

- Note that q measures how frequently shirkers are monitored and detected.
- If the firm uses highly effective monitoring technology q can become arbitrarily large.
- The probability of not being detected within τ is $e^{-q\tau}$
- If $q \to \infty$, $e^{-q\tau} \to 0$, so $P_{detected\ within\ \tau} = 1 - e^{-q\tau} \to 1$

Job Finding: a - From U to E

- **Definition**: Probability per unit time an unemployed worker finds a job.
  - A worker is unemployed at time t, the probability that they are employed at time $t + dt$ is $a \cdot dt$.
  - a is determined endogenously by the labour market conditions.
- **Steady-State Condition**:

$$b \cdot E = a \cdot U \tag{15}$$

- **Expression for a**:

$$a = \frac{NLb}{\overline{L} - NL} \tag{16}$$

- **Economic Implications**:
  - Reflects labour market tightness. $\uparrow a \Rightarrow$ jobs are easier to find.
  - Influences worker incentives to exert effort. $\uparrow a \Rightarrow \downarrow e$
  - Guides policy interventions to reduce unemployment.

Firm's Profit Function

$$\pi(t) = F\left(\bar{e}L(t)\right) - w(t)\left[L(t) + S(t)\right] \tag{17}$$

where $F'(\bullet) > 0$ and $F''(\bullet) < 0$.

- $F\left(\bar{e}L(t)\right)$: Total output, where $\bar{e}$ is the effort level and $L(t)$ is the number of workers exerting effort.
- $w(t)[L(t) + S(t)]$: Total wage cost, where all employed workers are paid.

Firm's Objective:

- Maximize instantaneous profits $\pi(t)$ by setting:
  - Wage w high enough to deter shirking.
  - Employment $L(t)$ at the profit-maximizing level.
- Higher wages reduce shirking ($S(t) \to 0$) but increase wage costs.

Final Assumption and Full Employment

$$\bar{e}F'\left(\bar{e}\frac{\bar{L}}{N}\right) \geq \bar{e} \quad \Longleftrightarrow \quad F'\left(\bar{e}\frac{\bar{L}}{N}\right) \geq 1 \tag{18}$$

For detailed explanation go to https://macroeconomics.info/.

- At full employment, each firm hires $\bar{L}/N$ workers.
- The implied Walrasian wage is $w^W = \bar{e}F'(\bar{e}\bar{L}/N)$.
- Workers supply effort only if $w^W \geq \bar{e}$.
- So productivity at full employment is high enough to cover effort cost.

Economic Insight:

- This is a benchmark assumption, not an equilibrium result.
- Without monitoring frictions, full employment is feasible under this condition.
- With imperfect monitoring, firms pay efficiency wages and involuntary unemployment can still arise.

Dynamic Programming

Key Idea: Use value functions to summarize the future in dynamic programming.

State Values:

- Let V_i denote the **value of being in state i**.
- States include:
  - E: Employment
  - U: Unemployment
  - S: Shirking
- V_i represents the **expected discounted lifetime utility** from the present moment forward of a worker in state i.

Why Constant?

- Transitions among states are **Poisson processes**.
- V_i is independent of how long a worker has been in a state (steady state assumption).

Deriving V_E - Analytically - 1/3

- In continuous time, time is divided into infinitesimally small intervals of length, Δt
- The Shapiro-Stiglitz model provides a framework to:
  - Evaluate the trade-offs between wages, effort, and unemployment risk.
  - Predict worker behavior (e.g., shirking).
- Let $V_E(\Delta t)$ and $V_U(\Delta t)$ denote the value of being employed and unemployed as of the beginning of an interval.
- Two components:
  - Utility from earning wages during Δt: $w - \bar{e}$
  - Future utility after Δt, accounting for:
    - Probability of remaining employed ($e^{-b\Delta t}$)
    - Probability of becoming unemployed ($1 - e^{-b\Delta t}$)

Deriving V_E - Analytically - 2/3

$$V_E(\Delta t) = \int_{t=0}^{\Delta t} e^{-(\rho+b)t}(w - \bar{e})\, dt + e^{-\rho \Delta t}\left[e^{-b\Delta t} V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right) V_U(\Delta t)\right] \tag{19}$$

For detailed explanation go to https://macroeconomics.info/.

Equation (19) has two parts:

1. $$\int_{t=0}^{\Delta t} e^{-(\rho+b)t}(w - \bar{e})\, dt$$
   represents the flow of utility from being employed during the interval $[0, \Delta t]$.

2. $$e^{-\rho \Delta t}\left[e^{-b\Delta t} V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right) V_U(\Delta t)\right]$$
   represents the expected utility from being employed or unemployed after Δt.

Deriving V_E - Analytically - 3/3

- If we compute the integral in equation (19) we get:

$$V_E(\Delta t) = \frac{1}{\rho + b}\left(1 - e^{-(\rho+b)\Delta t}\right)(w - \bar{e}) + e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right] \tag{20}$$

- Solving this equation for $V_E(\Delta t)$ gives

$$V_E(\Delta t) = \frac{1}{\rho + b}(w - \bar{e}) + \frac{1}{1 - e^{-(\rho+b)\Delta t}}e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right)V_U(\Delta t)\right]. \tag{21}$$

- Remember that $\boldsymbol{V_E = \lim_{\Delta t \to 0} V_E(\Delta t)}$ (Similarly for $\boldsymbol{V_U}$).
- To find this, apply L'Hôpital's rule to (21) then we get:

$$\boldsymbol{V_E = \frac{1}{\rho + b}(w - \bar{e}) + \frac{b}{\rho + b}V_U} \tag{22}$$

Deriving V_E - Intuitively - 1/2

- Think of employment as an "asset" that pays a stream of dividends over time:
  - **Dividends:** $(w - \bar{e})$. When unemployed, no dividends are received.
  - **Capital Loss:** $b(V_E - V_U)$. When unemployed, the asset loses value at rate b.
  - **The required return on the asset** ρV_E. The asset must provide a return of ρV_E per unit time.
- V_E is then the fair price of such an asset, which represents the **expected present value of all future dividends** a worker receives while employed, discounted at a required rate of return ρ.
- The expected return must thus be ρV_E per unit time.

$$\rho V_E = (w - \bar{e}) - b(V_E - V_U) \tag{23}$$

For detailed explanation go to https://macroeconomics.info/.

which is the same as equation (22). Rearranging gives the lifetime utility:

$$V_E = \frac{1}{\rho + b}(w - \bar{e}) + \frac{b}{\rho + b} V_U$$

Takeaway: Employment value reflects flow utility, transition probabilities, and discounting.

Deriving V_S and V_U - Intuitively - 2/2

- We can extend the asset analogy to the other two states.
- When a worker is shirking, the 'dividend' is w per unit time, but the rate at which he looses the job is also higher at $b + q$. Thus:

$$\rho V_S = w - (b + q)(V_S - V_U) \tag{24}$$

- When the worker is unemployed, he receives no 'dividend', but he gets a job ('capital gain') at the rate a, so

$$\rho V_U = a(V_E - V_U) \tag{25}$$

- We assumed above that if an unemployed worker gets a job, he exerts effort, as will indeed be the case in equilibrium.

Firms' problem

- A representative firm's profit per unit time is given (as in equation-17) by:

$$\pi = F\left(\bar{e}L\right) - w\left[L + S\right], \quad F'(\bullet) > 0, \quad F''(\bullet) < 0 \tag{26}$$

- L, S are numbers of workers who exert effort and shirk, respectively.
- The problem facing the firm is to incentivise employees not to shirk:
  - It must pay enough that $V_E \geq V_S$, otherwise workers prefer shirking.
  - The optimising firm will not overpay more then necessary and will choose w so the incentive constraints is just satisfied, i.e. $V_E = V_S$.
- Use $V_S = V_E$ in (24), and then subtract (23) and rearrange to get:

$$V_E - V_U = \frac{\bar{e}}{q} > 0 \tag{27}$$

- The workers thus must *strictly* prefer employment to unemployment.
  $\Rightarrow$ to induce effort, firms pay a premium over&above the cost of effort $\bar{e}$.

Wage level that induces effort

- Subtract ρV_U in (25) from ρV_E in (23) to get:

$$\rho(V_E - V_U) = (w - \bar{e}) - (a + b)(V_E - V_U) \tag{28}$$

- Substitute $\frac{\bar{e}}{q}$ for $V_E - V_U$ from incentive cond. (27) and solve for w:

$$w = \bar{e} + (a + b + \rho)\frac{\bar{e}}{q}. \tag{29}$$

- This is the wage level needed to induce effort, which
  - exceeds the cost of effort $\bar{e}$ by a positive amount.
  - increases in the cost of effort $\bar{e}$, the ease of finding jobs a, the rate of job breakup b, and the discount rate ρ.
  - decreases in the rate at which shirkers are detected q.
- The firms will pay this wage so there is no shirking in equilibrium.

The aggregate no-shirking condition (NSC)

- Since the economy is in a steady state, the number of unemployed workers is constant, so flows into and out of unemployment balance:
  - NLb workers become unemployed per unit time (where L is employment per firm and so NL is aggregate employment).
  - $a(\bar{L} - NL)$ unemployed workers find jobs per unit time.
  - Equating the two, we get the equilibrium job-finding rate as in equation (16):

$$a = \frac{NLb}{\bar{L} - NL}. \tag{30}$$

The no-shirking condition (NSC) - Reminder -

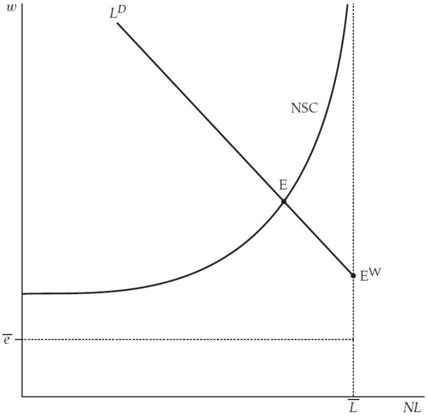
- Substitute (30) into (29) to get the **no-shirking condition (NSC):**

$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b\right) \frac{\bar{e}}{q}. \tag{31}$$

- No-shirking wage is an increasing function of aggregate employment:
  - When $NL \uparrow$, it is easier for unemployed workers to find jobs, see (16).
  - The cost of being fired falls, so wage has to rise to prevent shirking.

Equilibrium - 1/2 -

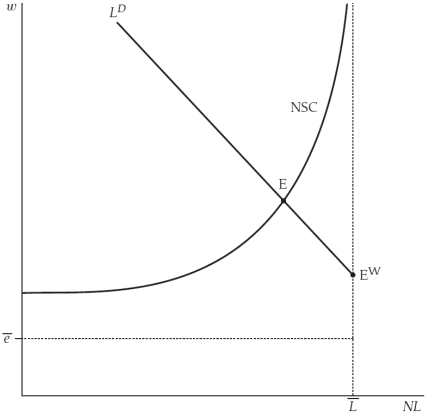
- The FOC of a firm's profit function (26) w.r.t. L yields
$$\bar{e}F'(\bar{e}L^*) = w, \tag{32}$$
so firms hire workers until marginal product of labour equals the wage.
- This implies downward sloping aggregate labour demand $L^D = NL^*$.
- In the absence of any monitoring issues, Walrasian equilibrium would occur at point E^W where L^D crosses the inelastic labour supply $\bar{L}$.
    - I.e. there would be full employment in equilibrium (assuming that the marginal product of labour at full employment exceeds cost of effort $\bar{e}$).



Equilibrium - 2/2 -

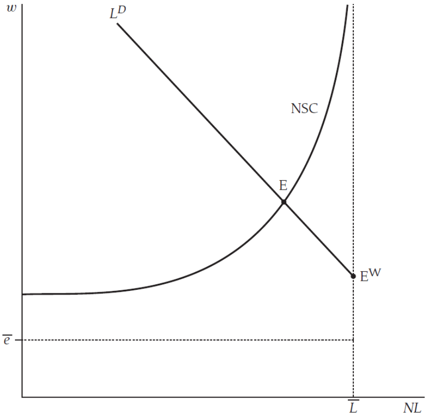
$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b\right) \frac{\bar{e}}{q}.$$

- However, with imperfect monitoring and possible shirking, equilibrium occurs at the intersection $\boldsymbol{E}$ of $\boldsymbol{L^D}$ and the no-shirking condition.
  - Wage is above the Walrasian level and there is positive unemployment.
  - Unemployed workers strictly prefer to be employed and exert effort,
  - but the wage does not fall, because then workers would start shirking.



A fall in labour demand in Shapiro-Stiglitz

- A negative demand shock shifts labour demand $\boldsymbol{L^D}$ left/down.
- New equilibrium is still on the NSC:
  - employment falls,
  - the no-shirking wage falls by less than in a Walrasian model.
- Key mechanism:
  - higher unemployment raises the punishment from job loss,
  - firms can sustain effort with a lower wage premium.
- Quantitative caveat: if NSC is steep at relevant unemployment rates, wage adjustment can still be sizable and employment responses modest.

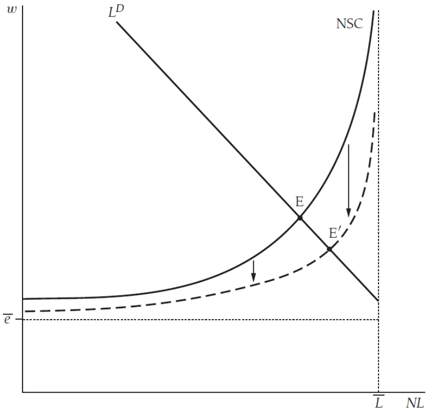


The effect of a rise in q

$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b \right) \frac{\bar{e}}{q}$$

An increase in the shirking detection rate q shifts the NSC down.

- The equilibrium wage falls and employment rises.
- Intuition: monitoring becomes better, so there is less need to incentivise workers by high wage.
- As $q \to \infty$, the economy approaches the Walrasian equilibrium.



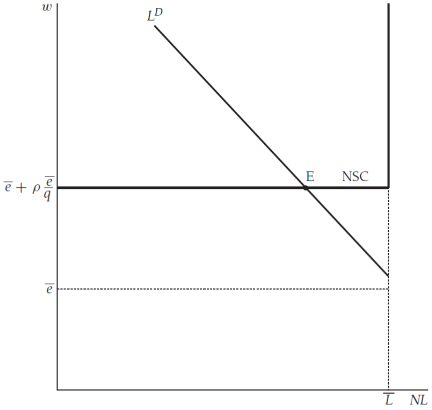
With turnover: $b = 0$

$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b\right) \frac{\bar{e}}{q}$$

If the job separation rate b falls to 0, there is no turnover, and unemployed workers are never hired.

⇒ The no-shirking wage in this case is simply $\bar{e} + \rho\bar{e}/q$, see (31)

- i.e. the NSC becomes flat and independent from employment.
- Intuitively, workers now only consider the cost of effort and the risk of permanently losing employment when contemplating shirking.



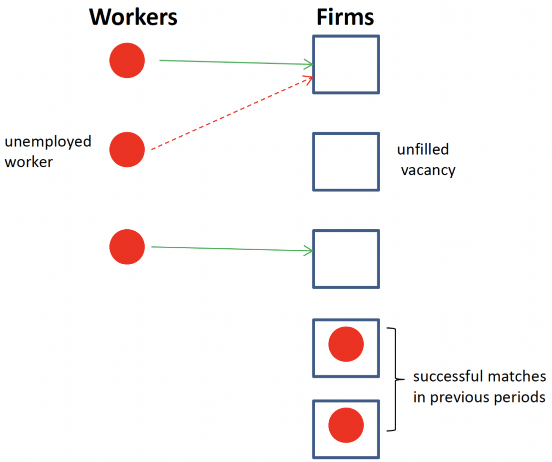
Extensions and limits of the model

- The framework is powerful but stylized.
- Extensions that matter for teaching and policy:
  1. **Hours vs layoffs**: with incentive problems, firms may prefer layoffs to work-sharing.
  2. **Dual labour markets**: high-wage monitored jobs can coexist with low-wage high-turnover jobs.
  3. **Alternative punishments**: penalties short of firing can flatten the NSC and change cyclical predictions.
- Open puzzle: if bonding/job-selling were frictionless, unemployment would be much lower.
- Interpretation: the model is best seen as a benchmark for incentive-based wage setting, not a complete labour-market description.

What have we achieved?

- Like any efficiency wage theory, Shapiro-Stiglitz model implies:
  1. There is involuntary unemployment.
  2. Although individual labour supply is inelastic when $w > \bar{e}$, shifts in labour demand result in movements along the relatively flat NSC curve. $\Rightarrow$ wages respond less to demand fluctuations, and employment more.
- But the formal model also yields additional insights:
  - The theory implies that **decentralized equilibrium is inefficient**, since the marginal product of labour exceeds the cost of effort. $\Rightarrow$ wage subsidies financed by lump-sum taxes improve welfare.
  - The model (modified to allow for flexible hours) also explains **why firms lay off workers** during downturns, rather than reduce hours. $\Rightarrow$ Reductions in hours make jobs less valuable, and hence workers would be more inclined to shirk.
- The view that workers are 'rational cheaters' is not uncontroversial – see homework for a more 'humane' theory due to Akerlof and Yellen.

One-Sided Search Model of Unemployment



Search and Unemployment

Search and Matching Models

Roadmap:

- Matching in the Labour Market
- The Supply Side: optimising Consumer/Workers
- The Demand Side: Optimising Firms
- Equilibrium
- The Beveridge Curve
- Experiments

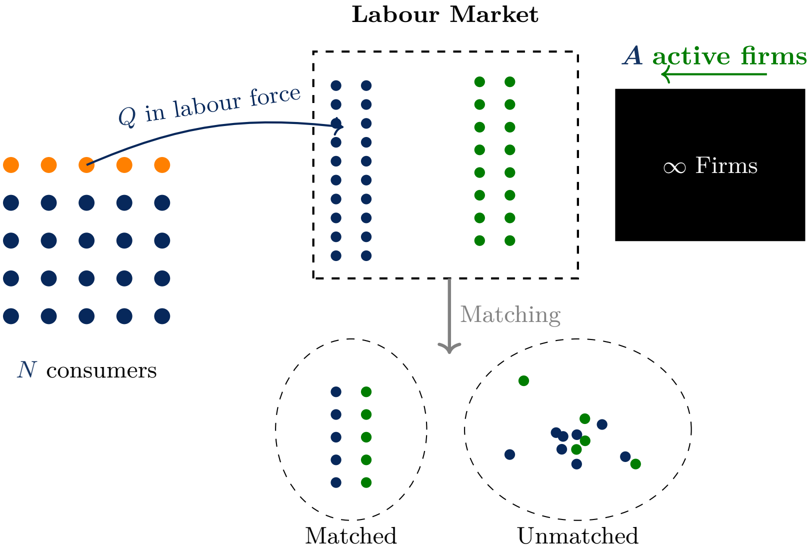
Search and Matching Models

This model is based on the framework introduced by Diamond (1982), Pissarides (1985), and Pissarides and Mortenson (1994)[1].

- The model is *two-sided*, incorporating both the supply side (consumers) and the demand side (firms)
- Workers and jobs are highly heterogeneous.
- Wages are determined by bargaining between workers and firms.
- Matching of workers and jobs occurs through a costly and complex process of **search and matching.**

1: Diamond (1982), Pissarides (2000), and Mortensen and Pissarides (1994) are seminal works in labour economics that led to their authors being jointly awarded the Nobel Prize in Economic Sciences in 2010. The prize was given to Peter A. Diamond, Christopher A. Pissarides, and Dale T. Mortensen "for their analysis of markets with search frictions". These economists developed what became known as the Diamond-Mortensen-Pissarides (DMP) model, which has become a fundamental framework for analyzing unemployment and labour markets.

Search and Matching

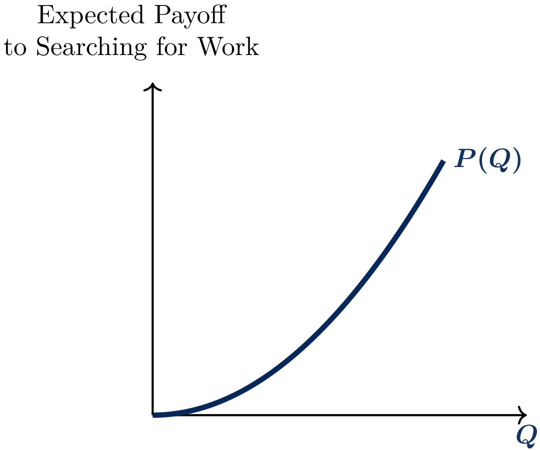


Terminology

- $\boldsymbol{N}$ denotes the working-age population.
- $\boldsymbol{Q}$ represents the labour force (that is, the total number of employed and unemployed individuals).
- $\boldsymbol{A}$ is the number of active firms that are searching for workers.
- The number of firms is determined endogenously within the model.
- $\boldsymbol{U}$ is the number of unemployed persons.
- Unemployment Rate: $\boldsymbol{U/Q}$
- Participation Rate: $\boldsymbol{Q/N}$

Workers

- Then, $N - Q$ represents those who remain in home production (i.e., not in the labour force: red area in the figure-5 above).
- We define a supply function, $P(Q)$, which represents the expected payoff from searching for market work.
- The supply curve $P(Q)$ is upward sloping because the opportunity cost of home production varies among consumers.
- A higher expected payoff from searching induces more consumers to forgo home production and participate in the labour market.



Firms

- In order to produce, a firm must post a vacancy to match with a worker.
- Posting a vacancy is costly; it costs the firm $\boldsymbol{k}$ (measured in units of consumption goods).
- Firms that do not post vacancies remain inactive and are unable to produce.

Matching and the Matching Function

- At the beginning of each period:
  - There are Q consumers actively searching for work.
  - There are A firms posting vacancies - searching for workers.

- Matching workers with firms is a time-consuming and costly process (heterogeneity).

- To capture these difficulties, we use a matching function.

- Let H be the number of successful matches (hiring) between workers and firms.

- Then ***the matching function*** is defined as

$$H = em(Q, A), \tag{33}$$

where:

  - e is an efficiency parameter

Matching and the Matching Function

$$H = em(Q, A)$$

- H: Number of matches (hires) | Q: Workers searching | A: Firms with vacancies
- $m(Q, A)$ is like a production function—which produces matches given inputs of workers searching and firms with vacancies.
- e: Matching efficiency (analogous to total factor productivity)
- **Reminder:** Higher e (via better search technologies/information) increases matches.

Properties of the Matching Function

The matching function $\boldsymbol{H = em(Q, A)}$ satisfies the following properties:

1. Non-negativity:

$$\boldsymbol{0 \leq em(Q, A)} \tag{34}$$

2. No matches if either input is zero:

$$\boldsymbol{em(0, A) = em(Q, 0) = 0} \tag{35}$$

3. Matches can't exceed the minimum of unemployed and vacancies:

$$\boldsymbol{em(Q, A)} \leq \min(Q, A) \tag{36}$$

4. Increasing but concave in inputs:

$$\boldsymbol{em_Q(Q, A) > 0}, \quad \boldsymbol{em_A(Q, A) > 0} \tag{37}$$

$$\boldsymbol{em_{QQ}(Q, A) < 0}, \quad \boldsymbol{em_{AA}(Q, A) < 0} \tag{38}$$

5. **Constant returns to scale**:

$$\boldsymbol{em(\lambda Q, \lambda A) = \lambda H} \tag{39}$$

The Supply Side: Probability of Finding Work

- **Probability of Finding Work (for a Consumer):**

$$p_c = \frac{H}{Q} = \frac{em(Q, A)}{Q} = e\,m\left(1, \frac{A}{Q}\right) \tag{40}$$

where $p_c \in (0, 1)$

  - Equation-40 use the constant returns to scale property of the matching function as in Equation-39.

- **labour market tightness** is defined as the ratio of firms with vacancies to job seekers.

$$j = \frac{A}{Q} \tag{41}$$

- The probability of finding work is given by:

$$p_c = e\,m\,(1, j) \tag{42}$$

The Supply Side: Probability of Finding Work

$$p_c = e\, m\,(1, j)$$

- By assumption, $m(1, j)$ is increasing in j, so a higher j implies a higher probability of finding work.
- The probability of finding work is increasing in the ratio of vacancies to unemployment.
- Consequently, **the probability of being unemployed** if a consumer chooses to search for work is

$$1 - p_c = 1 - e\, m(1, j) \tag{43}$$

where $(1 - p_c) \in (0, 1)$

  - $1 - p_c$ is decreasing in j
  - Higher labour market tightness implies lower probability of being unemployed. (many firms searching for workers, but few workers searching for jobs)
  - Higher labour market tightness implies higher probability of finding work. (many workers searching for jobs, but few firms searching for workers)

The Supply Side: Expected Payoff to Searching for Work

- Recall that $\boldsymbol{P(Q)}$ defines the supply curve for the number of consumers choosing to search for work, $\boldsymbol{Q}$.

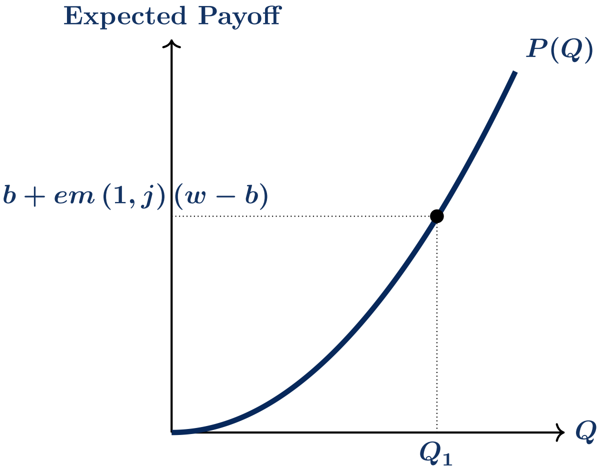
- In equilibrium, $\boldsymbol{P(Q)}$ must be equal to the expected payoff a consumer receives from searching, so
- If consumer work, they receive $\boldsymbol{w}$, (we will discuss later)
- If they don't work but actively search they still receive $\boldsymbol{b}$ which is the benefit they receive if they are unemployed.
- The expected payoff to searching is

$$\boldsymbol{P(Q)} = p_c w + (1 - p_c)\, b = \boldsymbol{b + em(1, j)(w - b)} \tag{44}$$

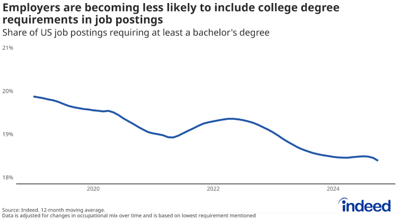
- The expression after the second equality is obtained by substituting for $\boldsymbol{p_c}$ using Equation-42.

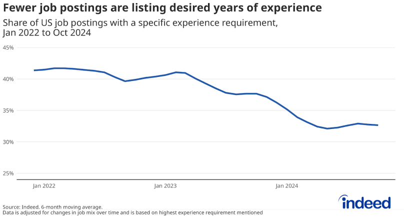
The Supply Side: optimisation by Consumers

- Figure is an illustration of Equation-44.
- The "market price" for searching workers, or the expected payoff to searching for work on the vertical axis, is determined by the market wage $\boldsymbol{w}$, the UI benefit $\boldsymbol{b}$, and market tightness $\boldsymbol{j}$.
- Then, given this market price, the supply curve for searching workers determines the quantity of searching workers $\boldsymbol{Q}$.
- A worker in a competitive equilibrium model observes the market wage and then decides how much labour to sell on the market at that wage.
- A worker also takes into account his or her chances of finding work and the unemployment benefit if his or her job search fails.

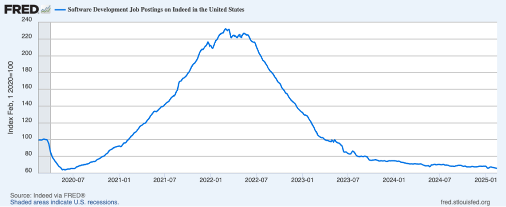


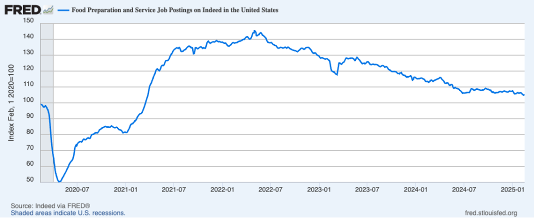
The Demand Side: Structural Shifts

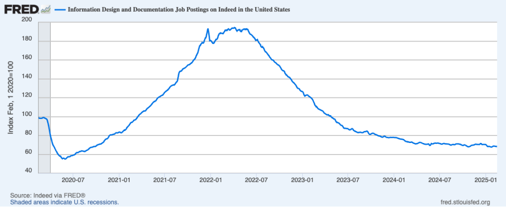


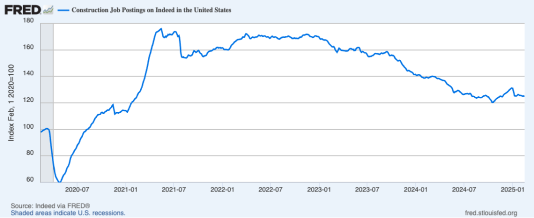


The Demand Side: Structural Shifts









The Demand Side: Probability of Filling a Vacancy

- **Remember:** Firms bear the cost k of posting a vacancy.
- **Probability of filling a vacancy:**

$$p_f = \frac{H}{A} = \frac{em(Q, A)}{A} = e\, m\left(\frac{Q}{A}, 1\right)$$

where $p_f \in (0, 1)$ and $j = \frac{A}{Q}$ is the labour market tightness.

- Then the probability of finding a worker or probability of filling a vacancy:

$$p_f = em\left(\frac{1}{j}, 1\right) \tag{45}$$

  - The probability of filling a vacancy for a firm is decreasing in labour market tightness j.
  - When the labour market is **tight** it is relatively easy for workers to find a job (i.e. p_f is high).
  - Conversely, when the labour market is **slack** it is easy for firms to fill a job, but difficult for workers to find a job (i.e. p_f is high and p_c is low).

The Demand Side: Expected Payoff to Posting a Vacancy

- When a firm is matched with a worker successfully, together they can produce output z where one firm and one worker produce z units of output.
- The firm and worker need to come to an agreement concerning the wage w that the worker is to receive.
- The profit the firm receives from the match is $z - w$, or output minus the wage paid to the worker.
- Firms will enter the labour market, posting vacancies, until the expected net payoff from doing so is zero, or

$$p_f(z - w) - k = 0$$

- Given Equation-45, we can write this equation as

$$em\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$

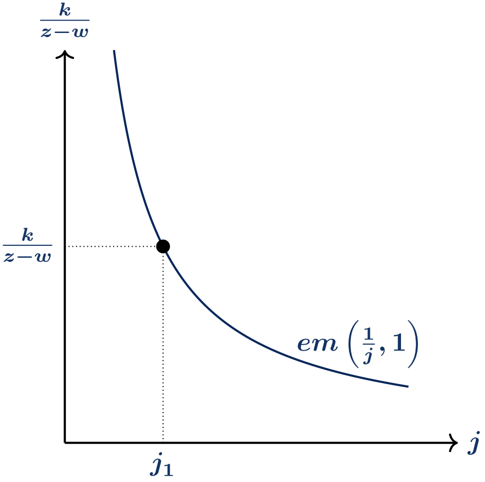
The Demand Side: optimisation by Firms

$$em\left(\frac{1}{j}, 1\right) = \frac{k}{z-w} \tag{46}$$

- This equation determines labour market tightness j, given the wage w, productivity z, and the cost of posting a vacancy k.
- Firms will enter the labour market and post vacancies, until the expected net payoff from doing so is zero, or

$$p_f(z-w) - k = 0$$

- This is depicted in Figure, where, given $k/(z-w)$, labour market tightness is j_1.



How to Share the Surplus? Let's Bargain!

- Successfull matches generate a surplus:

$$\textbf{Firm Surplus} = z - w \tag{47}$$

$$\textbf{Worker Surplus} = w - b \tag{48}$$

$$\textbf{Total Surplus} = (z - w) + (w - b) = z - b \tag{49}$$

- Need to determine the wage, but this is not a competitive market! In a competitive market, $w = z$.
- In a non-competitive market, the wage is determined by the bargaining power of the worker and the firm.
- **Solution:** Nash bargaining theory in this circumstance dictates that the firm and the worker will each receive a constant share of the total surplus based on their bargaining power.

Solution: Nash Bargaining!

- Let $\boldsymbol{a}$ denote the worker's share of total surplus, where $\boldsymbol{0 < a < 1}$. Here $\boldsymbol{a}$ represents the bargaining power of the worker.

$$\boldsymbol{w - b = a(z - b)} \tag{50}$$

- While the firm's share of total surplus is $\boldsymbol{(1 - a)(z - b)}$.

$$\boldsymbol{z - w = (1 - a)(z - b)} \tag{51}$$

- This is the solution to the Nash bargaining problem:

$$\max\, (w - b)^{a}\, (z - w)^{1-a} \quad \text{s.t.} \qquad \boldsymbol{Total\ Surplus = w - b + (z - w) = z - b} \tag{52}$$

- The solution to this problem is given by:

$$\boldsymbol{w = b + a(z - b)} \tag{53}$$

Equilibrium Wage

$$w = b + a(z - b)$$

Intuitions:

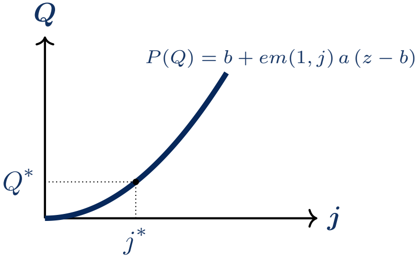
- The equilibrium wage is a linear combination of the output from a match, z, and the household's outside option, b.
- First, we require that $z > b$. If $b > z$, it would never make sense for households to search - the maximum wage they can earn is z, and if their outside option is greater than this, they would never choose to work.
- Second, if $a \to 1$, a firm has no bargaining power, and the equilibrium wage is $w = z$. But if this is the wage, the firm makes a negative profit of k by posting a vacancy.
- In contrast, as $a \to 0$, the firm has all the bargaining power, so workers are paid the minimum required to get them to search, which equals their outside option of b.

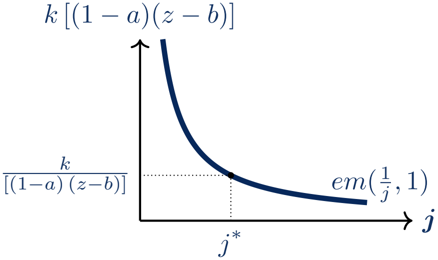
Equilibrium in the labour Market

$$\boldsymbol{em}\left(\frac{\mathbf{1}}{\boldsymbol{j}}, \mathbf{1}\right) = \frac{k}{z-w} = \frac{\boldsymbol{k}}{(\mathbf{1}-\boldsymbol{a})(\boldsymbol{z}-\boldsymbol{b})} \quad (54)$$

$$P(\boldsymbol{Q}) = b + em(1, \boldsymbol{j})(w-b) \quad (55)$$

- The first equation determines the equilibrium $\boldsymbol{j}$ given $\boldsymbol{a}$, $\boldsymbol{k}$, $\boldsymbol{z}$ and $\boldsymbol{b}$ (exogenous variables).
- Then, given $\boldsymbol{j}$ (endogenous variable), the second equation determines the equilibrium labour force - $\boldsymbol{Q}$.





Equilibrium Quantities

- **Unemployment Rate:**

$$\boldsymbol{u} = 1 - p_c = \mathbf{1} - \boldsymbol{em}\left(\mathbf{1}, \boldsymbol{j}\right) \tag{56}$$

- **Vacancy Rate:**

$$\boldsymbol{v} = 1 - p_f = \mathbf{1} - \boldsymbol{em}\left(\frac{\mathbf{1}}{\boldsymbol{j}}, \mathbf{1}\right) \tag{57}$$

- **Aggregate Output:**

$$\begin{aligned} Y &= H \times z = z \cdot em\left(Q, A\right) \\ \frac{Y}{Q} &= z \cdot em\left(1, \frac{A}{Q}\right) \\ \boldsymbol{Y} &= \boldsymbol{Q} \cdot \boldsymbol{z} \cdot \boldsymbol{e} \cdot \boldsymbol{m}\left(\mathbf{1}, \boldsymbol{j}\right) \end{aligned} \tag{58}$$

All together: Equations to keep track of

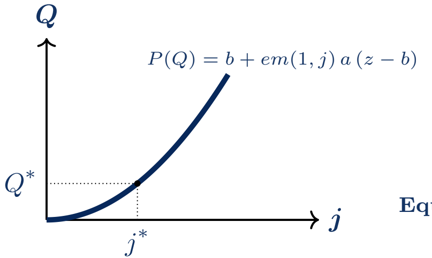
$$\textbf{Equilibrium Equations:} \begin{cases} em\left(\frac{1}{j}, 1\right) = \frac{k}{(1-a)(z-b)} \\ \\ P(Q) = b + em(1,j)a(z-b) \end{cases}$$

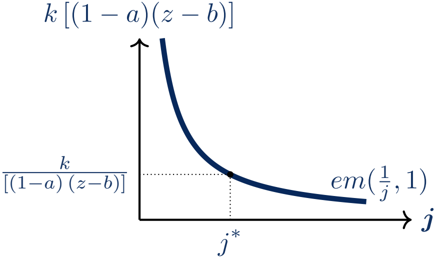
$$\textbf{Outcomes of Interest:} \begin{cases} u = 1 - p_c = 1 - em(1,j) \\ \\ v = 1 - p_f = 1 - em\left(\frac{1}{j}, 1\right) \\ \\ Y = Q \cdot z \cdot e \cdot m(1,j) \end{cases}$$

What happens if...

- Productivity, z, increases? **- AI revolution -**
- Efficiency, e, increase/decreases? **- Remote working -**
- Unemployment benefit, b, increases? **- European social state -**
- Cost of posting vacancy, k, decreases? **- Linkedin Effect -**

What would happen if: Experiments

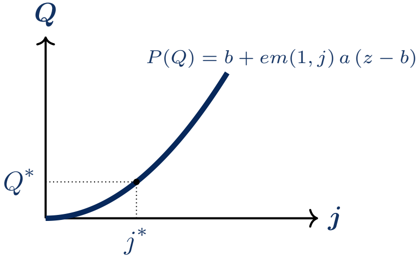


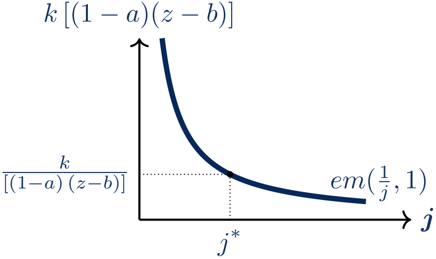


Equilibrium Equations:
$$\begin{cases} em\left(\frac{1}{j},1\right) = \frac{k}{(1-a)(z-b)} \\ P(Q) = b + em(1,j)a(z-b) \\ w = b + a(z-b) \end{cases}$$

Outcomes of Interest:
$$\begin{cases} u = 1 - p_c = 1 - em(1,j) \\ v = 1 - p_f = 1 - em\left(\frac{1}{j},1\right) \\ Y = Q \cdot z \cdot e \cdot m(1,j) \end{cases}$$

What would happen if: Experiments





- The smaller is the cost of posting a vacancy relative to the firm's share of total surplus, $\boldsymbol{k/(1-a)(z-b)}$, the higher job market tightness, $\boldsymbol{j}$ will be.
- If labour market tightness $\boldsymbol{j}$ is higher, then the chances of finding a job are greater for consumers, more of them will decide to search for work, and therefore $\boldsymbol{Q}$ will be higher.
- For example, in Figure higher $\boldsymbol{j}$ increases the expected payoff to searching for work, and then a higher supply of searching workers, $\boldsymbol{Q}$, is forthcoming.

The Beveridge Curve

Connecting Dots: The Beveridge Curve

- For simplicity, we are again going to focus on the **steady state**.
- U is constant, and flows into and out of unemployment balance:
  - $\lambda E = \lambda(1 - U)$ workers become unemployed per unit time, and
  - $p_c U$ workers find employment, allowing us to write:

$$\lambda(1 - U) = p_c U \tag{59}$$

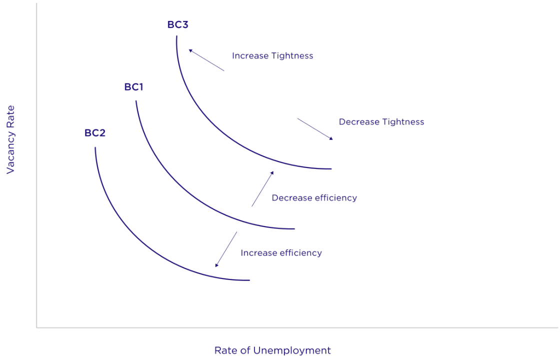
- Using that $p_c = em(1, j)$, where $j \equiv A/Q$, and rearranging yields:

$$U = \frac{\lambda}{\lambda + em(1, A/Q)} = \frac{\lambda}{\lambda + em(1, j)} = \frac{\lambda}{\lambda + p_c} \tag{60}$$

- Equation (60) defines a *negative* relationship between unemployment and vacancies, known the **Beveridge curve**.

The Beveridge Curve - PS5

- The Beveridge Curve compares the unemployment rate to the vacancy rate and shows how this changes over time.
- The Beveridge Curve is used to assess the current state of the labour market due to the economic cycle and is also a measure of the efficiency of labour market matching.
- Figure shows how movements along the curve will generally reflect cyclical changes in labour market conditions.
- For example, when the economy strengthens the unemployment rate will fall while job vacancies will rise. When the economy weakens, the opposite is true.
- Firms lay off workers, so unemployment rises and the number of vacancies falls.



From Beveridge Data to Structural Model

- The Beveridge curve is a reduced-form relationship between unemployment and vacancies.
- Data can show shifts, but not by itself **why** they occur.
- Next, we build a structural search-and-matching model to decompose:
    - job-finding vs separation forces,
    - wage bargaining effects,
    - vacancy-creation incentives.
- This gives policy-relevant comparative statics beyond visual curve shifts.